The longitudinal research project erStMaL, early Steps in Mathematics Learning, relates to the investigation of mathematical cognitive development in preschool and early elementary school ages from a socio-constructivist perspective. Within the framework of this project, an empirical family study (erStMaL-FaSt) is performed, which deals with the impact of the familial socialization for mathematics learning. The aim of the study is the development of theoretical insights in the functioning of familial interactions for the formation of children’s mathematical thinking. A basic concept is the “interactional niche in the development of mathematical thinking” (NMT), which will be adapted to the special needs of familial interaction processes. Catching up with the idea of MLSS (Mathematical Learning Support System) within this familial context, here it will be especially shed the light on the perspective, how elder sibling can be supportive or helpful for the learning process of a child.

INTRODUCTION

The IDeA (Center for Research on Individual Development and Adaptive Education of Children at Risk) is a research centre, which investigates the development of children at risk and the processes of individual learning. This research centre is constituted by the German Institute for International Educational Research (DIPF) and Goethe Universität Frankfurt. The financial support provided by the Ministry of Higher Education, Research and the Arts from the state of Hessen [1]. One of research project of IDeA Center is a Project erStMaL (early Steps in Mathematics Learning), which investigates the mathematical development of children with a specific emphasize on immigrant families. It is designed as a longitudinal study to follow children from the age of three until the third year of primary school. In the scope of the project erStMaL, a family study is performed, which is designed as a longitudinal study and named as erStMaL-FaSt (early Steps in Mathematics Learning-Family Study). The study deals with the impact of the familial socialization on the mathematics learning and due to the following three criteria, 8 participants are chosen from the project erStMaL. The criteria are the ethnic background (German or German/Turkish), the duration of the formal education of the parents and the sibling situation within the families (see Acar Bayraktar and Krummheuer 2011, Acar Bayraktar 2012). Data collection comprises of recorded videos and their transcripts. Once in a year, an appointment is arranged with each family, and therewith in total 3 observation phases are made. This leads step by step to a collection of data from each child. In each appointment the erStMaL child is video-recorded together with
members of the family while they are playing in different mathematical settings designed by the project.

For the family study two mathematical domains are chosen: Geometry and Measurement. Four play situations are conceived, due to these two mathematical domains. The members of the family are supposed to choose at least 2 games out of 4 and to perform them. For participation of all families, instruction manuals of each play are made both in German and Turkish, which can be spoken freely by families during play situations. The game materials are provided and put at the disposal of the family in the recording room.

### 2. THEORETICAL BASIS

“The play, for the child and for the adult alike, is a way of using mind, or better yet, an attitude toward the use of mind” (Bruner 1983, p. 69).

erStMaL-FaSt enables families freely to play with their children games with a mathematical background. During each play situation with maintenance of father/mother/sibling/grandmother the child explores something about the issued mathematical domain. This attendance of family provides to the child some “learning offerings” that are embedded in the interactive process of negotiation of meaning about the mathematical play. During the interaction of such various mathematical learning situations, there occur different emerging forms of participation and support. From a socio-constructivist perspective the cognitive development of an individual is constitutively bound to the participation of this individual in a variety of social interactions. During each learning action, each individual supports the development of each other through their own participation and interactive negotiation with each other. With respect of Bruner’s concept of a Language Acquisition Support System (LASS) we propose a similar concept for the learning of mathematics, which we call analogically the “Mathematics Learning Support System” (MLSS) (Bruner 1990). Tiedemann defines such a support system as a realization of patterns and routines in an adult’s and child’s interaction, in which the child is supported to participate in a mathematical discourse” (Tiedemann 2010, S.154; translation by Ergi Acar Bayraktar). In the family study the modes of functioning of MLSS in bilingual families are investigated. The analytical focus is set on the reconstruction of different types of support during the negotiation process in the mathematical play situations. Compared to works of Tiedemann (2012) and Bjorklund et al. (2004), in the family study it is encountered with different supportive activities of adults during the negotiation process with their children (cf. Acar Bayraktar 2014a, 2014b). With regard to the Bjorklund et al.’s supportive behaviour profiles of the adults the all-supportive activities in the family study are categorized and modified especially for the geometrical play situations through blocks (see Fig.1):
<table>
<thead>
<tr>
<th><strong>Supportive Activities</strong></th>
<th><strong>Description</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Prompt</td>
<td>Suggestion by the family member as to the generation of an answer without mention of use of any specific strategy. (e.g. How should you put the block?)</td>
</tr>
<tr>
<td>Prompt after error</td>
<td>Child makes error in calculation and family member prompts for a rebuilding. (e.g. Are you sure?, just look accurately at it!, be careful!, ok do it again!)</td>
</tr>
<tr>
<td>Affirmation</td>
<td>The family member demonstrates agreement to child’s answer or response to a math problem. (e.g. That’s right!, yes it has to be stand like that!, exactly!)</td>
</tr>
<tr>
<td>Disaffirmation</td>
<td>This is a type of correction, a definitive negative response indicating an incorrect response; disaffirmation could an explicit no. (e.g No, that is not right!; The corpus is wrong!; You built it wrong!; No, not like that!)</td>
</tr>
<tr>
<td>Provide solution</td>
<td>The family member provides the child with the correct solution. The family member spontaneously produces the answer.</td>
</tr>
<tr>
<td>Motivation</td>
<td>The family member motivates the child in a positive or negative way. It can be in different ways: by consoling, angering, encouraging, criticizing, insulting etc. (e.g. You are doing perfect!; In the next turn you will get better!; We have five rounds!)</td>
</tr>
<tr>
<td>Conclusion</td>
<td>The family member concludes the situation or gives commentary about the current situation. (e.g. Now it is dady’s turn!; You lose!, You get 3 points!; It can’t be played like that!)</td>
</tr>
<tr>
<td>Cognitive Directives</td>
<td>Three types of behaviours that direct the child’s thinking about the problem: modeling, instruction, re-representation.</td>
</tr>
<tr>
<td>Modelling</td>
<td>The demonstration of a strategy independent of instruction, that is, the family member models a behaviour for the child to observe and imitate (e.g. family members build the right or similar corpus in front of the player)</td>
</tr>
<tr>
<td>Instruction</td>
<td>The family member suggest the use of a specific strategy (e.g. Maybe you should put this block in between, or? ; When you take one block back, do you have the right corpus?; There are two blocks, on it- or?; There comes one more block up on it.)</td>
</tr>
<tr>
<td>Re-representation</td>
<td>When the family member re-represents the problem in a way that is more familiar to the child. (e.g. holding up blocks to represent to addend: one two three!)</td>
</tr>
</tbody>
</table>

Fig.1 Possible supportive activities of the family members during playing with blocks. (cf. original Bjorklund, Hubertz & Reubens 2004, p.351)
For the comparison among the various mathematical learning situations and for the longitudinal analyses, the concept of the “interactional niche in the development of mathematical thinking” (NMT) will be used, which has been introduced by Krummheuer (2011a, 2011b). He defines NMT as follows:

The concept of the “interactional niche in the development of mathematical thinking” (NMT), consists of the provided “learning offerings” of a group or society, which are specific to their culture and will be categorized as aspects of “allocation”, and of the situationally emerging performance occurring in the process of meaning negotiation, which will be subsumed under the aspect of the “situation” (Krummheuer 2012).

Due to the development of children’s mathematical thinking, Krummheuer and Schütte (2014) worked situational and allocation aspects of NMT up and associated them with the third aspect as “aspect of the child’s contribution”. Thereby NMT-Family is constructed as a subconcept of NMT, which offers the advantage of more close analyzes and comparisons between familial mathematical learning occasions in early childhood and primary school ages.

In view of the design of FaSt, three components of NMT-Family are shown and then their details given below:

<table>
<thead>
<tr>
<th>NMT-Family</th>
<th>component: content</th>
<th>component: cooperation</th>
<th>component: pedagogy and education</th>
</tr>
</thead>
<tbody>
<tr>
<td>aspect: allocation</td>
<td>mathematical domains: Geometry and Measurement</td>
<td>Play as a familial arrangements for cooperation</td>
<td>developmental theories of mathematics education and proposals of activeness for parents on this theoretical basis</td>
</tr>
<tr>
<td>aspect: situation</td>
<td>interactive negotiation of the rules of play and the content</td>
<td>leeway of participation</td>
<td>folk theories of mathematics education, everyday routines in mathematics education; MLSS</td>
</tr>
<tr>
<td>aspect: child’s contribution</td>
<td>individual actions</td>
<td>individual participation profile</td>
<td>competence theories</td>
</tr>
</tbody>
</table>

Fig 2. The structure of NMT-Family (Acar Bayraktar in press)

**Content:** In the practice of erStMaL-FaSt, children and their families are confronted with mathematical play situations, which are – as mentioned - either in mathematical domain “Geometry” or in mathematical domain “Measurement”. The play situations in erStMaL-FaSt are designed to offer the families opportunities for interactive negotiations. From the situational perspective, in these play situations, processes of negotiation emerge, in which the rules of play and/or mathematical topics might be chosen as themes. The focus (intriguing) child might contribute more or less actively.
to these negotiation processes. In such processes either different forms of efficient and original ideas can be brought and realized, or receptively the activities of other participants can be pursued (cf. Krummheuer & Brandt 2001).

**Cooperation:** The process of cooperation between the adult and child provides the opportunity to refine their thinking and to make their performance more effective. Depending on this cooperation, a different leeway of participation comes forward. “Leeway of participation” („Partizipationsspielraum“, Brandt 2004) is one of the interactionistic approaches, by which a child explores his/her cultural environment while co-constructing it. “Leeway” is taken here in the colloquial meaning of “room for freedom of action” (Krummheuer 2012). So, this is a concept belonging to the situational aspect. Brandt (2004) explains that the participants interactively accomplish different margins of leeways of participation that are conducive or restrictive to the mathematical development of a child. (see also Krummheuer 2011c; 2012). Alongside of contents, the children are involved in the social settings in the play situations, which are variously structured as in child-parents interaction and/or child-sibling interaction. These social settings need to be accomplished in the process of interaction. Hereby “Leeway of participation” of the focus (intriguing) child can be individually arisen out of the certain period of interaction process (cf. Brandt 2004, 2006). Therewith the child constitutes a definitive participations profile through his/her contribution.

**Pedagogy and Education:** Developmental theories and theories of mathematics education describe and delineate learning paths for the children’s mathematical growth from which point of view. With the respect to the folk pedagogy, the participating adults and children become situationally active and operant in the concrete interaction. The cognitive development of each individual is constitutively bound to the participation of these individuals in a variety of social interactions. During these interactions and participations in the mathematical discourses, there occurs a “support system”, which is proposed as a concept for the learning of mathematics and called “Mathematics Learning Support System” (MLSS) , with respect of Bruner’s concept of a Language Acquisition Support System (LASS) (Bruner 1986, p. 77; see also Acar Bayraktar and Krummheuer 2011, Krummheuer 2011b, Tiedemann 2010). In the manner of child’s contribution, learning process of the focus (intriguing) child might be characterized and thereby appropriate theories might be grasped under the title “competence theories”.

In the patterns and routines of interactions between child and families of family study, MLSS occurs in different ways. Through the negotiation of meaning adults and children try to share their knowledge by using either different type of explanations or statements or mathematical domains. While adults are imparting their knowledge, it is “usual” and “habitual” that they switch the mathematical topic from geometry to the arithmetic (Acar 2011). In the family study it is seen often that adults give “correct instructions” to their children at the arithmetic operations, while they “instruct” or “direct” their children “inappropriately” at the geometrical activities
(ibid). But yet, through all these negotiation processes, children and adults lay out new interpretations, which support the development of the child in a more or less constructive way (see Acar 2011, Acar Bayraktar and Krummheuer 2011, Acar Bayraktar 2012, 2014b).

With the respect to all these three components, it will be introduced one chosen scene as an example to show how the spatial abilities (spatial thinking) in the interactional niche in a familial context emerge.

**FAMILY GUL IS PLAYING “BUILDING 02”**

The mathematical play “Building 02” refers to geometry and spatial thinking. The family is supposed to build a three-dimensional version of the picture with wooden bricks, which all are in the same size and weight. Supposedly, they perform the relations between two- and three-dimensional representations. The player chooses one card from the deck and builds a wooden corpus related to the image on the card. In the play, cards are placed on the table face down. Each card has a difficulty level ranging from 1 to 4. The cards with the number 1 are the easiest. The cards with the number 4 are the hardest, whose transitions between the various blocks are fluid and are purposed more complicated.

![Fig 3. The game cards in different levels](image)

The required information about Family Gül for the chosen situation is in the following table:

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Berk</strong></td>
<td>7 years old</td>
<td>Primary School</td>
<td>German, Turkish rudimental</td>
</tr>
<tr>
<td><strong>Elder Brother</strong></td>
<td>13 years old</td>
<td>Secondary School</td>
<td>German, Turkish</td>
</tr>
<tr>
<td><strong>Grandmother</strong></td>
<td>Studied 5 years</td>
<td>Lower Education</td>
<td>German rudimental, Turkish</td>
</tr>
</tbody>
</table>

![Fig 4. Information about Family Gül](image)

For the coming analysis, a scene from the play “Building 02” is chosen, which is materialized in the second observation phase. In the chosen scene Berk’s game partners are his elder brother and the grandmother—will be called granny—. In total, they play 10 rounds by turns. The transcribed scene is from the second round, which begins with Berk and ends up with granny. For the transcription a part of Berk’s turn is chosen.
Fig 5. Recording position and the chosen card

At the beginning of chosen scene, Berk shows the card to his game partners and then starts to build the figure immediately up. He puts vertically first three blocks (K1, K2, K3) next and parallel to the each other. Then he puts two blocks (K4, K5) bonded to each other horizontally on these three blocks. Thereafter he centres a block (K6) on the conjunction of both blocks K4 and K5, vertically in a same direction with K1, and sets another block (K7) up on K6 horizontally in a parallel direction to K4 and K5. Therewith he completes the built corpus (see fig. 6) up till chosen transcription. The built corpus that in the upright position on the table stands is showed as follows and thereafter comes the transcribed scene:

Fig 6. Built corpus by Berk with the named blocks and sides from the front view

Transcript  [2]

16  Brother:  *wrong*, looks and smiles at granny
17  Granny:  is it wrong? looks and smiles at Berks’ brother
18  Brother:  nodes and looks at Berk by laughing
19  Granny:  you did it wrong, laughing looks at Berk
20  Brother:  wrong, laughing looks at Berk
21  Berk:  *holds the chosen card and looks at it why?*
22  Brother:  wipes the surface of blocks K4, K5 from the corpus this is in the middle and there are three of it. *knocks it down*

When the chosen card and the built figure are compared, it can be said that the built corpus resembles with the figure on the chosen card. Berk situates all blocks with their narrow sides as is seen in the figure on the chosen card (see Fig. 7). Thus the faces of blocks are congruent both in the figure and the corpus, which makes the corpus resemble to the figure on the chosen card.
Fig 7. The chosen card and the built corpus with the named blocks and sides from the front view, and the wide side (X) and narrow side (Y) of building blocks

However in the middle part of the figure on the chosen card there has to be three horizontal blocks (cf. d, e, h), which are marked with the red line (see fig. 7). Apparently Berk built this part of the corpus a bit different; there is a resemblance between both blocks K4 and e or h, and K5 and d or e. Nevertheless in the middle of the Berk’s corpus (see Fig. 7) there are only two blocks instead of three. This can be either a ‘gestalt’ problem (1) or ‘static balance’ problem (2) of Berk (see fig. 8):

Fig 8. ‘gestalt’ of the chosen card and the built corpus

1. Berk might pay attention only on the gestalt of the figure on the chosen card. Thus the gestalt of the figure resembles with the gestalt of the corpus (see Fig. 8). By decomposing the figure in his mind and composing each pieces of this figure as a 3D corpus might be an adaptation problem from the pieces to the whole.

2. Berk might pay attention only the static balance between the blocks. Clements and Samara point out that children balance blocks intuitively, but often they place blocks off centre (Clements & Samara 2007, p.512). Instead of situating each block on one block, he might prefer to situate K5 on two blocks (K3 and K1), and K4 on two blocks (K1 and K2) (see Fig. 8), on which both blocks might be in static balance and might not have a falling hazard.

In both case, it can be said that the built corpus resembles to the figure on the chosen card. Looking from a developmental perspective, Berk is able to represent block sides at the detailed level but topologically he might have either an adaptation problem by
decomposing and then composing shapes or spatial problem by regulating spatial relations of the objects and figures. From a participatory point of view, Berk is one of the game partners and he is coming up totally with a new idea by building a resembling corpus that he seems like an autonomous person [3].

The brother smiles and tells Granny that Berk built the corpus wrong <16>. From a supportive perspective his reaction can be called a disaffirmation, which is a type of correction, a definitive negative response indicating an incorrect reaction. Thereon granny smiles at him and poses a question if Berk built the figure wrong <17>. Brother replies her by nodding <18>. From a participatory point of view the elder brother takes of the 'standart role' as a 'mentor' [4], which was rather reserved for his granny by telling that the built corpus is wrong. Moreover granny seems to accept his 'mentoring' by taking this idea up with him. From a supportive perspective the reaction of elder brother might be interpreted that he nodes in agreement to granny’s question. Within this context his reaction might be assumed as an affirmation. In another regard his reaction might be seen as a disaffirmation, that he indirectly responses to Berk’s action indicating an incorrect reaction, which might be a type of correction as supportive action. Thereby the elder brother gravitates into using two different support types at the same time.

Thereafter granny declares that Berk built the corpus wrong <19>, while she and his brother go on to laugh at Berk <15-19>. Thereby she uses the issue of wrongness of Berk’s corpus, which elder brother has already stated in the line <16>. She might take the idea of her grandson, that Berk built the corpus wrong, and might formalize the same idea in a same way as elder son. Thereby she might keep on focusing on the same point as elder grandson. From supportive perspective her reaction can be seen as a 'disaffirmation', that she indicates an incorrect response.

The elder brother declares by laughing that the built corpus is wrong <20>. This reaction might be seen as a confirmation of grandmother’s declaration. Therewith he keeps his idea still constant and from participatory of view keeps his role by telling again that Berk built the corpus wrong. From the supportive perspective he still determines on the wrongness of built corpus, by using the support type 'disaffirmation’. During this disaffirmation process, the brother and granny laugh at Berk, which can be interpreted in different ways: They might laugh at Berk to

1. make him think that he builds the corpus wrong.

2. show that he lost the game. It can be such a gratification, that Berk made a mistake and lost his turn.

3. distract his attention away from the building activity in order that he constructs the corpus wrong.

4. hide their own stress that he builds the corpus right.

Berk holds the chosen card, looks at it and asks “why?” <21>. Here it seems that Berk checks the chosen card, to find out the mistake between the figure on the card
and the built corpus. After looking at it, he poses this question, maybe to experience either the reason of their commentaries or the reason of incongruence. The older brother gives him a feedback immediately and wipes the surface of blocks K4, K5, K6 from the corpus. Then he says, “this stands in the middle and there are three of it”<22-23>. His answer can be interpreted in different ways:

- “This is in the middle”: “In the middle” could be mentioned either the facet of blocks d, e, h in the figure or K4 and K5 the facet of blocks in the corpus, which are shown with the red lines on the figure 4. In all probability, he might compare block e or block h in the figure with the block K4 in the corpus.

When the figure and the corpus are compared, it can be seen that one block is missing in the built corpus. Thereby Brother’s explanation “This is in the middle” could be interpreted:

1. “This block (K4) has to be normally between the blocks d and h, so that K4 could be in the middle of corpus”. If one more block were not missing in the corpus, then the block K4 would have been in the middle of the corpus. Thus, block e in the figure 4 (see fig. 4) represents the block K4 in the corpus. Herewith the missing block in the corpus is identified with the block h in the figure.

2. “This block (e) is in the middle of the corpus. Here it has to be one more block in the middle.” If this block were not missing in the corpus, then between the blocks K4 and K5 would have been one more block in the middle of the corpus. Thus, block e (see fig. 4) represents the missing block, while K5 represents block d and K4 represents block h in the corpus. Herewith the missing block in the corpus is identified with the block e in the figure.

- “There are three of it”: This explanation can be interpreted as follows:

3. “There are three blocks (d, e, h) in the middle of the figure (see fig. 5). It means there has to be also three from each block in the corpus.”

4. “The block (K4) has to be in the middle of the corpus thus in total there has to be three blocks. But here there are two. Actually there has to be three blocks.”

5. “There has to be three blocks and K4 has to be in the middle. But Berk used only two blocks”.

6. “The block (e) (see fig. 5) is in the middle of the figure. At the figure there are three blocks in the place, which is red lined in the Figure 4. But there are two in the corpus. Actually there has to be three blocks.”

- “This is in the middle and there are three of it”: the brother might mean that, the block K6 stands in the middle of the corpus and there has to be two more blocks same as K6, then they are all together three.

From the participatory point of view the elder brother keeps his role autonomously [3], by constituting so far his declarations and disaffirmations <16-20>. Thereon he
determines the facts during showing them on the corpus. In this sense, he steals the ‘standart role’ as a ‘mentor [4]’ from granny by establishing Berk’s action and behaves as a mentoring adult. From a supportive perspective the reaction of elder brother can be called an instruction, by which he suggests the use of any specific strategy as a supportive activity. However, the given instruction of elder brother seems that it comprises sufficient knowledge on geometry but unsufficient knowledge on pedagogy. By conveying his thoughts, the elder brother only gives Berk the geometrical hint starkly but not pedagogically that he does not take the note of Berk’s comprehensiveness. Therewith his mentoring occurs in such a way, which can be seen as technical and operational.

Thereafter the elder brother bowls over the built corpus. His reaction might show that he assumes that Berk’s turn is over. Thereby he behaves like that Berk has not rejected his argument and herewith it seems that the discussion about the rightness of the built corpus ends up non-ambiguously. In this sense there might be improvisely a working consensus between granny, elder brother and Berk, which means they don’t keep on discussing about the built corpus. From participatory point of view, he circumscribes leeway of participation of Berk by terminating Berk’s turn and possible further discussions about the wrongness of the built corpus. From supportive perspective brother’s reaction can be seen as a conclusion that Berk built the corpus wrong and his turn is over.

**SUMMING-UP**

The chosen scene is the second round of the game. The chosen scene begins with Berk’s turn, in which a polyadic interaction process emerges. He shows the card to his game partners and then starts to build up the figure. After he is done with the corpus, his brother comes with the comment that the built corpus is wrong and imposes his argument on granny that she stands up to the wrongness of built corpus too <16-20>. Berk asks the reason of this argument and his brother replies him that there has to be actually one block, which stands in the middle between two blocks, and thereby there should be three blocks in the middle of the built corpus <21-23>. After his explanation the brother cleans up the corpus and Berk’s turn ends. Thereby there emerges improvisely a consensus between them so that Berk built the corpus wrong.

According to chosen scene the elder brother takes a role as a mentor, which is usually unexpected of the ‘standard’ social-constructivist approaches. In such approaches, the adult takes over the standard role as a “mentor” of the child’s learning process. In the chosen example, which is actually ‘unusual’ for the ‘standard’ social-constructivist approaches, the elder brother improvises the role as a ‘wise adult’ rather than granny, while she is mostly taking the role as an over-hearer (Krummheuer 2011c, pp. 83-84) in the play situation. Vandermaas-Peeler highlights that the older siblings tend to scaffold and encourage younger children’s play (2008, p.282). In Berk’s turn the elder brother gives instructions, feedbacks or clues instead of granny, which can be
interpreted as that kind of behaviour in the sense of Vandermaas-Peeler of scaffolding and encouraging. Additionally it also seems that the brother pays attention to the building activities of Berk. This also reinforces the idea that the elder brother takes the role as a ‘wise person’.

**REFLECTION OF THE NMT-FAMILY AND CONCLUSION**

“The play under the control of the player gives to the child his first and most crucial opportunity to have the courage to think, to talk, and perhaps even to be himself” (Bruner 1983, p. 69).

According to foregoing analysis the three components of an interactional developmental niche in familial context can be reconstructed as in follow:

**Component “Content”:** Block Building provides a view of children’s initial abilities to compose 3-D objects. In the chosen play, four goals are pursued: Spatial structuring, operating shapes and figures, static balance between blocks, identifying the faces of 3-D shapes to 2-D shapes. By National Research Council is also reported that 5-years-old children can understand and can replicate the perspectives of different viewers. These competencies reflect an initial development of thinking at the relating parts and wholes level (National Research Council 2009, p.191). Berk realizes the spatial relations between 2-D and 3-D objects. He can relate some parts with the whole. So, as an allocation aspect occurs the spatial structuring and operating with shapes during the play. As a situational aspect, Berk negotiates with his elder brother and granny. In this trial structure, there emerges a negotiation about the built figure. There occurs a consensus, that Berk built the figure incorrectly. As an aspect of contribution, through his active and autonomous [3] participating Berk can directly apply the static balance thereby he can explore its idea by building the robust corpus. Furthermore he examines at the same time the resembling gestalt of the figure and corpus.

**Component “cooperation”:** The chosen scene comprises two turns in one round, in which polyadic interaction process emerges. Elder brother mostly directs the play situation and assists Berk briefly to let to explore him how the figure actually has to be. However the course of interaction went in such a way that Berk’s leeway of participation was limited that he cannot discuss later on the wrongness of the built corpus with his elder brother or granny. Therefore, from the situational aspect, Berk has a circumscribed leeway of participation.

From the aspect of contribution, Berk presents his own idea during building process of the corpus. Therefore he seems like an autonomous person [3]. Moreover he tries to negotiate with his elder brother about the built corpus, which enables him to be an ‘active participant’.

**Component “pedagogy and education”:** The chosen play situation refers to the spatial structuring in geometry. In the chosen scene, by the organizing and setting objects, the graphical- and spatial-development of Berk are slightly assisted.
As an allocation aspect, the chosen scene refers to explore and examine the spatial structuring and visualizing.

As a situational aspect, there occurs a supportive activity as disaffirmation, through which instantly elder brother directs the play situation by his technical and operational mentoring. Thereby he provides Berk to confront with his mistake.

As an aspect of contribution, Berk experiences building a robust corpus and getting similar gestalt with the figure on the card. Thus it seems that he has strong spatial abilities and by his active participating he can examine the idea of the static balance.

As a consequence of these three components, NMT of Berk is constructed as follows:

<table>
<thead>
<tr>
<th>NMT Family Gül Building 02</th>
<th>component: content</th>
<th>component: cooperation</th>
<th>component: pedagogy and education</th>
</tr>
</thead>
<tbody>
<tr>
<td>aspect of allocation</td>
<td>Geometry, Spatial structuring, operating shapes and figures, static balance between blocks, identifying the faces of 3-D shapes.</td>
<td>Playing with brother and granny</td>
<td>Theory of the development of spatial skills and spatial structuring</td>
</tr>
<tr>
<td>aspect of situation</td>
<td>negotiation between brother, granny and Berk about the built figure; consensus</td>
<td>circumscribed leeway of participation</td>
<td>Through his disaffirmation, elder brother provides Berk to confront with his mistake. technical and operational mentoring</td>
</tr>
<tr>
<td>aspect of contribution</td>
<td>exploring static balance to build the robust corpus; examining the resembling gestalt of the figure and the corpus.</td>
<td>Active participant: ‘autonomous’</td>
<td>Having strong spatial abilities building a robust corpus and getting similar gestalt with the figure on the card</td>
</tr>
</tbody>
</table>

Fig 9. The structure of NMT-Family Gül in the chosen Scene of the play “Building 02”

In the chosen scene granny takes the role as an over-hearer and enable to the elder brother being a ‘mentor’. However the elder brother is geometrically ‘wise person’ yet pedagogically unsufficient person that he gives Berk the geometrical hints starkly but not pedagogically that he does not take the note of Berk’s comprehension.
Therewith he mentors Berk in technical and operational ways. In the sense of the functioning of MLSS (Mathematical Learning Support System), granny and elder brother realize a support system for Berk through these interaction patterns and routines.

In terms of Berk, by being mentored by an elder brother and confronting with his mistake, Berk becomes able to “see” the wrongness of the built corpus. Therewith a potential developmental niche for Berk might emerge. Furthermore the ‘unusual’ mentoring by elder brother might enable Berk to improve and enhance his spatial abilities. Clements and Samara reports that spatial processing in young children is not qualitatively different from that of older children or adults, but children with the age produce progressively more elaborate constructions (2007, p. 512). In this sense spatial processing of Berk can be enhanced and he can produce more elaborate constructions with his age and through his brother’s mentoring.

Hence, It will be really exciting to go on augmenting the examples of Berk, and to find out how NMT-Family functions work on Berk’s spatial development in his familial context in over situations.

NOTES
2. Rules of Transcription
   Column 1 Serially numbered lines.
   Column 2 Abbreviations for the names of the interacting people.
   Column 3 Verbal (regular font) and non-verbal (italic font) actions.
   underlined Speech is in Turkish
   standard Speech is in German (regular font)

   The sides of block are defined as X Side, Y Side, Z Side in transcript.

4. From the socio-constructivist perspectives parent has a ‘standart role’, who is one of the most important figures in a child’s life, serving as a mentor, a model; providing aid for the child (for more see Bruner 1986, 1996, 2002; Rogoff 1984,1990; Sfard 2001, 2008; Tiedemann 2010,2012 and also Acar Bayraktar & Krummheuer 2011). In this sense, the definition ‘mentoring’ are used as a term for the ‘standart’ and ‘inherent’ parental guiding, whose boundaries can be differentiated at each parent.

REFERENCES


