MATHEMATICALLY CREATIVE PROCESSES IN EARLY CHILDHOOD

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The paper deals with mathematically creative processes in early childhood. Therefore the concept of the interactional niche in the development of mathematical creativity is introduced, which combines interactionistic theories of socio-constructivism with sociocultural theories and a psychoanalytically based attachment theory to describe mathematically creative processes of children under the specific perspective of early childhood development. Data are collected in the interdisciplinary project MaKreKi (mathematical creativity of children), in which researchers from mathematics education and psychoanalysis examine the development of mathematical creativity of children in the age range of 4-8 years.

INTRODUCTION

Definitions of mathematical creativity differ with respect to several assumptions. On the one hand creativity referred to as the individual ability of a person in the sense of divergent thinking (Guilford, 1967), the abilities to produce fluent, flexible, novel and elaborated solutions to a given problem (Torrance, 1974) or the ability to produce unexpected and original work, that is adaptive (Sternberg & Lubart, 2000). On the other hand creativity is seen as embedded in a social process (e.g. Csikszentmihalyi, 1997; Sriraman, 2004; Vygotsky, 2004), in which creativity is not solely located in a person's cognition, but is also accomplished in the social interaction among members of the society.

My research interest is the examination of mathematical creativity in early childhood under the specific perspective of early childhood development. In this contribution I focus on the social and sociocultural approach to creativity. The first section presents a theoretical approach that deals with the question in which forms of social interactions these early mathematically creative interactions of children are evoked and supported. Afterwards, based on these theoretical assumptions, I define what I understand as a mathematical creative process in early childhood. Finally I clarify which cultural and sociocultural dimensions should be considered in a theory of mathematical creativity in early childhood, and in what way. After that an overview of data collection and methods is given. Next an empirical case follows. The paper ends up with summary and prospect.

THEORETICAL APPROACH

In many fundamental works on children's creativity, play has been regarded as a social situation in which creative actions arise and eventually are fostered (e.g. Bateson & Martin, 2013; Vygotsky, 2004,). Also from a psychoanalytical perspective play has been regarded as a location where children's creativity is formed (Winnicott, 2012). According to Winnicott play is neither part of the personal inner reality nor part of the actual external reality but of a third dimension, that he indicates as "potential space" (Winnicott, 2012, p.144). As soon as a child experiences his or her mother no longer as part of his or her own, a "playground" (Winnicott, 2012, p. 64) emerges, that the child can use for creative activities. This complex process depends highly on a supportive mother, who is willing to participate and to reciprocate. Winnicott calls her the "good enough mother" (p.109), who is sensitive and reacts appropriate to her child's needs in opposition to the "not good enough mother" (p.109).

Developing a theory of mathematical creativity in the early years one has also to consider the development of mathematical thinking as well. From a sociocultural perspective children's play is also considered as location of the development of mathematical thinking (e.g. Carruthers & Worthington, 2011; van Oers, 2002) and for this reason the development of mathematical creativity, too, is at least implicit. Mathematics comes into play through articulation by more knowledgeable people, "their companions in the cultural community" (van Oers 2002, p. 30). If play involves other more knowledgeable persons like children or adults, opportunities for scaffolding (Bruner, 1986) or guided participation (Rogoff, 2003) emerge. Besides the aspect of children's play with competent partners, children are dealing with mathematics in their free self-initiated play as well as in play situation with peers (e.g. Carruthers & Worthington, 2011).

Mathematical creative processes in early childhood

In several publications of the MaKreKi-project we have shown that "non-canonical" (e.g. Münz, 2014) solving processes can be seen as mathematically creative processes. They include the following aspects (Krummheuer, Leuzinger-Bohleber, Müller-Kirchof, Münz, & Vogel, 2013; Sriraman, 2004):

Combinational play: Under this aspect the accomplishing of unusual combinations of insights and experiences and the sense of playfulness in the manipulation of procedures and its transfer to new areas are understood. With reference to Finke (1990) these activities are summarized with the "combinational play" (p. 3) of framing a mathematical situation.

Non-algorithmic decision-making: According to Ervynck (1991), mathematical creativity articulates itself when a unique and new way of problem solving emerges. For the age group of interest, processes of problem solving can be new, creative and unique, although they are not new for the mathematical community. Uniqueness can be seen as the "divergence from the canonical" (Bruner, 1990, p. 19) way of solving a mathematical problem in early childhood, that adult mathematicians would not necessarily expect.

Adaptiveness: Sternberg and Lubart (2000) characterize creativity as the ability to create an unexpected and original result that is also adaptive to the given real situation. We have redefined this concept to our specific needs. *Adaptiveness* in

MaKreKi describes children’s ability to accomplish unusual definitions of situations and to convince their partners by their alternative framing of the situation. So a mathematical creative action has to be reasonable, which mean there are arguments, why the chosen combination or unusual definition of the situation leads to a mathematically correct solution (e.g. Lithner, 2008). Additionally these arguments have to be somehow mathematically funded (e.g. Lithner, 2008; Münz, 2014).

Social and sociocultural dimensions

As already mentioned a theory of mathematical creativity has to consider social and sociocultural dimensions of creativity, because creative behavior can be seen as intertwined in a complex person-situation interaction. I introduce a framework, which stresses these interactive structures and in which the emerging *creative process between* children is regarded as an aspect of interactive *process* of negotiation of meaning between the involved persons.

Developing a theory of mathematical creativity in the early years one has also to consider the development of mathematical thinking as well. According to my research focus, the “concept of the *interactional niche in the development of mathematical thinking (NMT)*” of Krummheuer (2012) seems appropriate for several reasons. For a start the NMT describes the situational aspect in the development of mathematical thinking as a process of negotiation of meaning between the involved persons. So it allows the description of this negotiation of meaning in a creative process as well and is able to capture insights about this creative process. Additionally another procedural aspect is described in the form of the cooperation of the involved persons and the individual scope of action of a child. The NMT is a further development of the original components of the “developmental niche” of Super and Harkness (1986), who describe it

> … as framework of studying cultural regulation of the micro-environment of the child, and it attempts to describe the environment from the point of view of the child in order to understand processes of development and acquisition of culture. (p. 552).

Additionally Krummheuer added the aspect of interactively local production of such processes, which includes besides the aspect of allocation (under which the provided mathematical activities of a group are summarized, see table 1) the aspect of situation (situationally emerging accomplishment occurring in the process of meaning making) and the aspect of action (which covers the individual contributions to the actions as well as the individual participation profile of a child). This approach can be adopted to the theory of mathematical creativity in early childhood to examine the mathematical creative *process*. To describe a mathematical creative *process* in early childhood, in the following the fourth line of Krummheuer’s NMT “aspect of action” (Krummheuer & Schütte, 2014) is renamed as the *aspect of individual’s creative action*. It highlights the used mathematical concepts by the individual child, which can be regarded as *combinational play*, *divergence from canonical* and *adaptive*, as well as the individual profile of participation of the child (see table 2).

	Component: Content	Component: Cooperation	Component: Education/pedagogy
Aspect of allocation	Mathematical domains; Bodies of tasks	Institutions of education; Settings of cooperation	Scientific theories of mathematics education
Aspect of situation	Interactive negotiation of the theme	Leeway of participation	Folk theories of mathematics education
Aspect of action	Individual contributions to actions	Individual profile of participation	Competency theories

Table 1: The NMT of Krummheuer (Krummheuer & Schütte, 2014)

Krummheuer has furthermore divided these aspects into three components: ***content***, ***cooperation*** and ***education/pedagogy*** (Krummheuer & Schütte 2014), which I briefly summarize and emphasize their relevance to a theory of mathematical creativity in early childhood:

Content: The following data are collected in the MaKreKi-project, in which on the level of allocation mathematical topics are usually designed as mathematical situations of play and exploration (Vogel, 2013), regarding the children's assumed mathematical competencies. They offer opportunities for children to demonstrate their mathematical creative potential. An assisting adult, who can be seen as a more knowledgeable person, presents the situations. On the situational level this presentation generates processes of negotiation. The presentation of the mathematical situations of play and exploration and the processes of negotiations lead to individual mathematically creative actions.

Cooperation: Children participate in culturally specific social settings, which are variously structured as in peer-interaction or small group interaction guided by a more knowledgeable person. These social settings do not succeed immediately. They need to be accomplished in the joint interaction. Depending on each event, a different "leeway of participation" (Brandt, 2004, p.47) of the children will come forward. By embellishing these possibilities of participation every child has an individual "profile of participation", which can be relatively stable over a given time (see Brandt, 2004, p.47).

In the limited frame of the paper I will not focus on the component ***education/pedagogy***, but for the sake of completeness, in this column Krummheuer investigates the influence of scientific as well as folk theories of mathematics education on the development of mathematical thinking (Krummheuer & Schütte, 2014).

Psychoanalytical dimensions in mathematically creative processes

To integrate psychoanalytical insights about creativity in early childhood I add another column, the component *interpersonal relations*, that derives from Winnicott's concepts of the "good enough mother" (Winnicott, 2012, p.109) as a requirement for the "potential space" (p. 64, p. 135) as the origin of creativity in human life, which I added in a third column (see table 2).

Interpersonal relations*:* According to Winnicott the initiation of playing is associated with the life experience of the baby who has come to trust the mother figure (Winnicott, 2012), which is given when she reacts sensitively and warmly to the child's needs. In the first years of life the child develops an 'inner working model' through child-parents-interactions (Bowlby, 1969). This 'inner working model' contains the early individual bonding experiences as well as the expectations, which a child has towards human relationships, derived from these experiences. They induce the child to interpret the behavior of the caregiver and to predict his or her behavior in certain situations. After the first year of life this 'inner working model' becomes more and more stable and turns into a so-called "attachment pattern" (Bowlby, 1969, p.364). At this time the child has developed mental representations in which the caregiver is seen as an independent, intentionally dealing object. Nevertheless the inner working model of the child encompasses especially own motives and experiences in attachment relationships. The child does not yet regard that the caregiver has it's own plans, motives, experiences and emotions, too, which may differ from the child's ones. This ability develops at preschool age und leads to the possibility of child and caregiver to develop a relationship, which Bowlby terms a "partnership" (Bowlby, 1969, p.267). The beginning partnership looms when children are able to integrate the plans of the caregiver in their owns and try to influence them. Usually this starts at the age of three. At the age of four children show another behavior, which expresses the begin of a new attachment relationship, that is called the goal-corrected partnership (Benson & Haith, 2009, p.34). According to Bowlby (1969) and Marvin & Britner (1999) the basis of this new partnership is a cognitive and a communicative ability: The ability of the child to gain insights about the goals and emotions of the caregiver and to coordinate these on a representational level (perspective taking) and as well as in case of conflict between plans of child and caregiver to negotiate a common plan with the caregiver. The children are now able to see two or more representations as components of a higher plan. In the relationship with the caregiver they can represent their own plans as well as the plans of the caregivers simultaneous but separated, which allows them to compare both perspectives to see if they coincide or if they have to develop a common perspective.

The quality of the child-caregiver relationship in sense of the attachment patterns can be measured (Ainsworth, Blehar, Waters & Wall, 1978). A rough distinction can be made between two types: The secure and the insecure attachment pattern. Children with a secure attachment pattern have, thanks to their sensitive mothers, a chance to build up secure relationships to their mothers in which the whole spectrum, of human feelings in the sense of communication with each other can be perceived, experienced and expressed. Children with insecure attachment patterns experiences a mother who shows no intense affects and behaves in a distanced controlled manner or who sometimes reacts appropriately, and at other times is rejecting and overprotective, on the whole, inconsistent in a way that is unpredictable for the child. Empirical observations of infant's exploratory behavior as well as children's play behavior point out, that children with a secure attachment pattern show more exploratory behavior more positive affect, and are more cooperative in their play than children with an insecure pattern (e.g. Creasy & Jarvis, 2003). Also the quality of the play seems to depend on the attachment pattern. Following Crowell and Feldman (1988), parents of children with insecure attachment pattern are focusing on basic task completion rather than on learning processes.

On the level of allocation of a NMC (see table 2) this attachment pattern can be regarded as stable (Bowlby, 1969), nevertheless Bowlby suggests that these attachment patterns may change as the child begins to interact with other attachment figures e.g. siblings, peers, teachers (Bowlby, 1969). So on the level of situation a child with an insecure attachment pattern may meet other children or adults who show sensitivity to his or her needs in the sense of being a "good enough" partner (Winnicott, 2012, p. 109), which enhances his or her potential for cooperation during the interactive process and potential in mathematical activities. The reverse conclusion is also conceivable. Regarding the aspect of individual creative action, the "good enough partner" will accept the contributions of the child to the mathematical process and understand it's perspective in framing the mathematical situation. Therefore the child has also to realize and accept the different perspective of it's partner in the concrete situation and to use communicative strategies in the negotiation of a common perspective. The acceptance has not only to be understood as a shared meaning, it can also be seen as an interim, in which the involved persons have squared their framings of the situation and concluded that there is more than one possibility of framing. If this is not the case, the creative process somehow fails in the concrete situation. The following table summarizes the additions to the concept of NMT, which I term as the "interactional niche of the development of mathematical creativity" (NMC):

	Content	Cooperation	Interpersonal Relations
Aspect of allocation	Mathematical domains; Bodies of tasks; mathematical potentials	Institutions of education; Settings of cooperation	Attachment Patterns of the involved persons
Aspect of situation	Interactive negotiation of the theme	Leeway of participation	Situational emerging of attachment patterns
Aspect of individual's creative action	Individual mathematically creative actions	Individual profile of participation	Acceptance in form of shared meaning or interim

Table 2: The NMC

EMPIRICAL APPROACH & METHODOLOGY

The sample of MaKreKi is based on the original samples of two projects that are conducted in the "Center for **I**ndividual **De**velopment and **A**daptive Education of Children at Risk" (IDeA) in Frankfurt, Germany. One project is a study of the evaluation of two prevention programs with high-risk children in day-care centers (EVA). It examines approximately 280 children. The second project is a study of early steps in mathematics learning (erStMaL). This project includes approximately 150 children. Thus the original sample contains 430 children. We asked the nursery teachers of the two original samples, whether they knew children in their groups who show divergent and unusually sophisticated strategies while working on mathematical tasks. We could identify 37 children, who seem to work creatively on mathematical problems.

For the examination of the development of mathematical creativity in the selected children, we introduced mathematical situations of play and exploration (Vogel, 2013) constructed in the erStMaL-project. They are designed in a way that the children can demonstrate their mathematical potential in the interactive exchange with the other participants. An assisting adult is supposed to present the material by sparingly giving verbal and gestural impulses. To ensure that the implementations of these mathematical situations are independent of the participating individuals and proceed in comparable ways, the mathematical situations of play and exploration are explicitly described in forms of design patterns of mathematical situations (Vogel, 2013). With respect to the longitudinal perspective, the children are observed every six months while they work on two mathematical situations of play and exploration. All these events are video taped with two cameras.

For the diagnosis of the attachment pattern we apply the Manchester Child Attachment Story Task, so-called MCAST (Green, Stanley, Smith & Goldwyn, 2000). This is a story telling test that has good reliability and validity.

Regarding the theoretical considerations and the attempt to identify mathematically creative moments in mathematical interactions of preschool children, in the following there is an analysis of interaction conducted, which is based on the interactional theory of learning mathematics (Brandt & Krummheuer, 2001). It focuses on the reconstruction of meaning and the structuring of the interaction process. Therefore it is proper to describe and analyze topics with regard to contents and the negotiation of meaning in the course of interactional processes. The negotiation of meaning takes place in interactions between the involved people.

The applied analysis of interaction is derived from the ethnomethodologically based conversation analysis, in which among others it is stated that the partners co-constitute the rationality of their action in the interaction in an everyday situation, while the partners are trying constantly to indicate the rationality of their actions and to produce a relevant consensus together. This is necessary for the origin of their own conviction as well as for the production of conviction with the other participating persons. This aspect of interaction is described with the term "accounting practice" Lehmann, 1988, p. 169). To analyze these "accounting practices" of children in mathematical situations, the reconstruction and analysis of argumentation of Toulmin (1969) have proved to be successful.

VICTORIA AND SINA IN THE "SOLID FIGURE-SITUATION"

Information about the girls and the "Solid Figure-Situation"

Victoria (4;10 years old) and Sina (4;6 years old), both have a secure attachment pattern, participate in the "Solid-Figure-Situation". They are close friends.

In the mathematical situation of play and exploration "Solid-Figure" the attending children deal with geometrical solids: Cube, square column, pyramid, triangular prism, cone, cylinder and sphere, each of them is duplicated. The material and the designed impulses of the assisting person provide a geometrically mathematical content for the children in which they are getting to know these solids and their properties. To enable the children to focus their attention more easily on the geometric figures, a little bag in which the children feel them is used.

"…because this is a gyroscope"

At first the female assisting person (abbreviated B in the following) calls on both girls to handle the cone and describe what they have touched. Afterwards she puts a red cube, a red pyramid and a blue cylinder on the table. The children grab and term each solid unrequested: The cone has been designated as "castle" by Victoria and "hat" by Sina, the cube as "cube", the cylinder as "gyroscope" and the pyramid has been identified as "cornflake" by the group. In this context Sina inquires if they have to build a castle and begins to put some solids on top of each other. B negates and presents a little bag, in which she has put a cylinder, what the children have not seen. She invites the girls to find out, which solid is located in this bag only by touching and gives the clue that the solid in the bag is also arranged on the table. Therefore every girl has to feel the unknown solid in the little bag. Victoria starts and says: "A gyroscope. Next Sina follows and says: "A gyroscope, too". After validating their conclusion initiated by B the girls are adding together some solids to build towers.

First they put two cylinders on top of each other and place the pyramid on the cube to see which one is the tallest tower. Then Victoria asks: “Should I fetch the yellow one?” and Sina answers: “Okay. Good. Come on.” and Victoria continues: “Then we can look which one is bigger”. Victoria places the cone and the pyramid side by side. Immediately thereafter Sina puts the pyramid on top of the cylinders and the cone on the cube and than interchanges cone and pyramid (Figure 3).

Figure 3: Towers

B asks: “Which one was the biggest Victoria? Have you seen it so fast?” And Victoria puts her right hand on the pyramid and says: “This one.” Whereupon B responds: “Put them down again. Than you can look again which one is bigger”. But Victoria looks at Sina’s towers and says by grabbing the cube-pyramid-tower: “Or Sina no. Do you know what? These ones belong to these ones“. And by touching the cone on top of the cylinders she notes: “And these to these ones”. Sina moves the double cylinder-cone-tower towards the cube-pyramid-tower and tells: “Bigger”. B looks to Victoria and wants to know why these solids belong together whereupon Victoria shrugs her shoulders and Sina responses: “This is red and red”, by touching the pyramid and the cube. And Victoria continues: “Yes because and look and this belongs to the gyroscope because this is a gyroscope”, she points at the double cylinder-cone-tower. Then she holds both hands at the cube-pyramid-tower, “because it“, and turns her hands and shows her palms.

Analysis of the episode of Victoria and Sina in the “Solid Figure-Situation”

Regarding the component **content** on the level of *allocation* B provides a geometrically mathematical content for the children in which they are getting to know solids and their properties. The task is to find identical solids. On the *situational* level this content is extended to build castles or towers (Sina), to find out which solid is the biggest one by direct size comparisons (Victoria) and to find similar solids, which belong together (Victoria). These extensions lead to the *mathematical creative action* of Victoria by grouping pyramid and cube as well as cylinder and cone together. Her *combinational play* in matching cylinder and cone is interpreted as the assignment of the solids regarding similar properties. This similar properties are expressed in Victoria’s terming of both solids as “gyroscope” (in German “Kreisel”). The german word “Kreisel” includes the word “Kreis” which is translated in English “circle”. Cylinder and cone have both circular base areas. In an analogous manner Victoria matches cube and pyramid, which she constitutes in showing her palms, which can be portended as a non-verbal addressing of the base areas of the solids, because like cylinder and cone, cube and pyramid have also similar base areas. Victoria's assignment of the solids can be regarded as a conclusion, *divergent from the canonical*, because the intended task of B was to find identical solids. Her conclusion seems *adaptive* to the group, because no one disagrees. However B invites her to explain her findings, which she does by using a plausible and mathematical underpinned warrant in emphasizing the same base areas of the solids, which has also a plausible and mathematical backing (solids with one equal property can be grouped together).

Regarding the topic **cooperation**, Victoria and Sina are paired in a dyad together with an assisting person in their day-care center. They have to work on a task consecutively, because B addresses first Victoria and then Sina to feel the unknown solid. Summarizing B has the role of an *initiator* of tasks and *evaluator* of solutions while the girls have to process that task as *processors*. They are not allowed to build a castle. The polyadic changes to a more dyadic interaction structure between B and one girl, whereas Sina's referring to Victoria's solution "A gyroscope, too" focuses a maintaining of the polyadic interaction. B's initiation of an evaluation of the solution addresses both girls. After completing B's task the roles of the girls change. They are focusing Sina's idea of building towers and they *initiate* new tasks autonomously like the comparison of sizes. They realize a dyadic interaction between each other. B behaves more reserved her role shifts to a *facilitator*, who inquires, like her inviting of Victoria to say which solid is the biggest one. It seems somehow as she is one step behind the girls. At that time Victoria is able to conduct two dyadic discourses simultaneously between her and B and her and Sina. In the second one Sina puts cone and cylinder together as well as cube and pyramid. Victoria comments these activities with the idea that these solids belong together. Again B inquires further information, she asks Victoria for an explanation of these groupings. Sina's expression "Bigger" may refers to B's first question concerning the size comparison and her explanation of the grouping "This is red and red" answers B's second question, so again she focuses a maintaining of the polyadic interaction. In the polyadic discourse Victoria extends Sina's explanation with the evidence of the same base areas of the matched solids. So she can be seen as the *initiator* of this explanation why the matched solids belong together. Summarizing the girls' role changes from a *processor* to an *initiator* and the dyadic interaction structure turns to a polyadic ones.

Concerning the component **interpersonal relations** both girls have a secure attachment pattern, which mean that they show high exploratory behavior and cooperative strategies in their play. This has also been showed in the presented episode. In the concrete situation Sina shows efforts in maintaining a polyadic interaction. Furthermore she supports Victoria in her idea of comparison of sizes ("Okay. Good. Come on."). Also Victoria supports Sina's idea of building castles or towers although B has rejected it. The scene shows how Victoria and Sina cherish in their idea of building castles. They first response on B's question and afterwards immediately they begin to raise their questions and build castles. In terms of

Winnicott Victoria has in Sina a “good enough” partner. Later on B can also be seen as a good “enough partner”, because she shows interest in Victoria’s conclusion and invites her to explain her findings. In the polyadic interaction a mathematical “playground” (ibid., p. 64) emerges, that enables Victoria to accomplish a mathematically creative process. The conclusion of grouping pyramid and cube as well as cone and cylinder together can be interpreted as a shared meaning between the three persons. But both girls have different explanations: Sina argues with the same colour, Victoria with the same base area. But Victoria understands and bears her explanation as an extension of Sina’s one, she starts with “Yes because and …”. So both explanations do not be mutually exclusive, but have equal rights. Both perspectives of framing the (mathematical) situation are legitimate in the group.

SUMMARY AND PROSPECT

Understanding mathematical creativity in early childhood as a cooperative process, that emerges in the situational negotiations of meanings in social interactions the concept of the niche of the development of mathematical creativity has been introduced and demonstrated on an empirical case. It highlights the allocative and situational terms of a mathematically creative process as well as the individual mathematically creative action.

As the example shows, from the socio-constructivist view a mathematically creative process arises in the concrete situation once a not anticipated interpretation of a mathematical situation occurs in the interaction between the involved interlocutors. Although both girls follow the instruction of the assisting adult, which differs from her own interpretation of the situation, they do not forget their own plans in building castles. So first they meet the adult’s invitation and her expectation in answering which solid is hidden in the bag. Then they reorganize the social order of the interactional process by raising own questions and building castles. These reorganizations are necessarily for the ongoing interactional process in which Victoria accomplishes her mathematically creative action.

Victoria and Sina have extended the mathematical content “geometrical solids and their properties” and “finding identical solids” to “build towers“, “size comparison” and “finding similar solids”. These changes in the mathematical content are linked to a changing of their social participation structure. The girls’ roles alter from *processor* to *initiator*. From a psychoanalytic point of view both girls show high cooperative strategies in supporting each other’s ideas, which may be linked to their attachment patterns. Also B’s role changing as response to the children’s needs seems to support the mathematical creative process of Victoria in grouping the mentioned solids together because of their same base area. This conclusion occurs in the polaydic interaction, as Victoria has to explain her grouping in a deep mathematically funded argumentation. It will be interesting to analyze how Victoria’s mathematically creative processes develop when she gets older and enters school.

Concerning children with insecure attachment patterns, the question arises how they behave in mathematical situations of play and exploration. How develop their mathematically creative processes and do they have different profiles of participation due to their attachment patterns? Additional analysis and niches may give some answers.

The niche of the development of mathematical creativity has not been completed by now. Additional analysis of scientific and folk pedagogy's concepts about mathematical creativity and mathematics education arising in the social setting can give further insights about the conditions of the development of mathematically creative processes.

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