In the Head of an Upper Secondary School Student

Mathematical Thinking and Reasoning

September 2013

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ABSTRACT

The aim of this study is to describe how and when during the course of my teaching the students develop their mathematical thinking and reasoning. The study will focus on whether or not the students can account for the development of their own abilities in mathematical reasoning and thinking. Through a selection of mathematical Activities we have done, the study will focus on the learning outcomes for the students by using qualitative methods with interviews. The main group of students studied in this project has been the pre-IB class at Áva Gymnasium during the academic year 2012/2013. There is also a smaller group of students who are enrolled in the Mathematical Studies or the Standard Level Mathematics course. There were a total of nine interviews with the students in pre-IB and three interviews with the students in their second IB Diploma Programme year.

The results reveal that the students think that all of the Activities were good, but that there are no outstanding tasks. These tasks helped to create diversity and variation instead of monotonous routine work. The students in IB2 mean that they were better motivated by working with the Activities, because the tasks let the students succeed and feel satisfied with themselves, to recognize their own mathematical capabilities and to help them think, reflect and gain more mathematical knowledge. The students in the pre-IB year didn’t fully comprehend the mathematical Activities used during their first year and they could not yet express their reflections on their learning. Homework assignments and tests were also important for students’ learning. Many students learned a lot from the discussions we had during the Activities.

*It takes time* to develop the students’ mathematical thinking and reasoning. It is a gradual process which is helped immensely by working with well-chosen mathematical Activities on a long term basis. The students must learn to dare to take on a challenge and be responsible for their learning. As a teacher, I must dare to work with tasks where I can’t predict what will happen, and I need to listen to the students’ reasoning and thinking. All three students in IB2 had changed their attitude to mathematics completely, and recognized their own mathematical capability.
ACKNOWLEDGEMENTS ........................................... 4

1 INTRODUCTION .............................................. 5
1.1 Background .................................................. 5

2 PROBLEM AND PURPOSE ................................. 7
2.1 Purpose ....................................................... 7

3 LITERATURE RELEVANT FOR THIS STUDY ............ 8
3.1 Definitions ................................................... 9

4 METHOD .......................................................... 11
4.1 Technology .................................................... 11
4.2 Group size ..................................................... 11
4.3 Ethics applied in the classroom ............................ 12
4.4 Activity one – The Gradient ............................... 12
4.5 Activity two – Little Boxes ............................... 13
4.6 Activity three – Generalizations in Geometry .......... 14
4.7 The Interviews ................................................. 16

5 RESULTS AND ANALYSIS OF THE INTERVIEWS ..... 18
5.1 The interviews with pre-IB students ...................... 18
5.2 Activity - The Gradient ....................................... 18
5.2.1 Analysis of the interviews about the Gradient ....... 20
5.3 Activity - Little Boxes ....................................... 21
5.3.1 Analysis of the interviews about Little Boxes ....... 22
5.4 Activity – Generalizations in Geometry .................. 23
5.4.1 Analysis of the interviews about Generalizations in G .. 25
5.5 The interviews with IB2 students ................................. 26
5.6 Student One .......................................................... 26
    5.6.1 Analysis of Student One ................................. 27
5.7 Student Two .......................................................... 27
    5.7.1 Analysis of Student Two ................................. 28
5.8 Student Three .......................................................... 29
    5.8.1 Analysis of Student Three ................................. 31

6 SUMMARY/DISCUSSION ............................................. 32

7 REFERENCES ............................................................ 35

8 APPENDICES ............................................................ 36

ACKNOWLEDGEMENTS

I wish to express my genuine gratitude to all those who have helped me in the process of writing this project. Many, many thanks to all my students in IB12 and IB10 at Äva Gymnasium and also to my colleagues who have helped me with constructive input to my work. In particular I would like to thank my supervisor at Malmö Högskola, Eva Riesbeck, for her valuable advice on how to perform the project and analyse the results. Eva-Stina Källgården also gave me such good constructive input, advice and interesting discussions about theories of learning. Most of all, thank you Carol Adamson! Without your critical comments on prior versions of this paper and the time spent with me in supporting my thinking and writing, this work wouldn’t have been possible.
1 INTRODUCTION

How do students perceive how they learn mathematics through problem solving? For more than six years I have been developing and including mathematical Activities in my teaching. I base an entire lesson on one mathematical Activity. It might be a problem-solving task or focus more on understanding a mathematical concept or procedure. We work and discuss the problems as a class or we split the class in smaller groups. Some sessions require a graphing calculator and some sessions don’t.

My experience is that most of my students developed more positive attitudes to mathematics and a deeper appreciation for mathematical thinking, which in turn led to better examination results. In their individual evaluations, students claimed that this was a better way for them to understand mathematics. They thought it was more fun and they learned much more.

By listening to my students and observing them to try to understand their thinking, I wanted to find out in this study how and when during the course of my teaching the students developed their mathematical thinking and reasoning. Could the students themselves account for the development of their own abilities?

1.1 Background

My methods of teaching mathematics have changed enormously since I started to work as a teacher in 1986. During the last six years I have been using more investigative approaches, more open problems and other suitable means to give the students the pleasure of discovering mathematical connections or to discuss meaningful mathematics. The intent of my teaching during the first of the three years the students study at my school is for each student to gain a much more firmly based understanding, as well as greater enjoyment and motivation in mathematics. A new learning/teaching approach requires them to set new expectations. The majority of the students, who had been taught traditionally, would need to change their expectations the most. Therefore it was necessary to go slowly in the beginning when Activities were introduced to make sure that the students gained as much as possible from these lessons.

Sfard (2001) means that we are not yet fully aware of the importance of mathematical conversation for the success of mathematical learning. She believes in a communicational approach to the study of human cognition and that thinking may be conceptualized as a result of communication. Learning mathematics can be defined as an initiation to mathematical discourse, or initiation to a special form of communication known as mathematical.
Some of the aims in all mathematics courses in the IB programme are to enable students to:

- enjoy mathematics, and develop an appreciation of the elegance and power of mathematics
- develop an understanding of the principles and nature of mathematics
- communicate clearly and confidently in a variety of contexts
- develop logical, critical and creative thinking, and patience and persistence in problem-solving
- employ and refine their powers of abstraction and generalization

The students in my pre-IB classes come with very mixed abilities in mathematics. The majority of them (approx. 70%) come from Swedish lower secondary schools. The others attended International Schools or public schools abroad. Almost 70% of the students are native Swedish speakers. There are very few native English speakers in the class.

At Åva Gymnasium the students are enrolled for three years, the first an IB preparatory year. During this first year they study the Swedish National Maths Courses labelled Ma 1B, 1C and part of 2C, as well as other subjects in English. During the following two years they are enrolled in the International Baccalaureate Diploma Programme if they pass the courses in pre-IB.

The IB Diploma Programme is designed as an academically challenging and balanced programme with final examinations that prepares students for university. The programme is normally taught over two years and has gained recognition and respect from the world's leading universities. In Sweden there are 32 Upper Secondary Schools that offer the IB Diploma programme and there are more than 2400 IB Diploma schools worldwide.
2 PROBLEM AND PURPOSE

The aim of this study is to describe how and when during the course of my teaching the students develop their mathematical thinking and reasoning. When during their first year at the Upper Secondary School do students realize that their mathematical reasoning and thinking is changing? Can they account for the development of their own abilities?

Other questions to consider are: What changes the students’ attitudes to maths? Are they better motivated by the Activities we do? Is it the use of the technical devices (Graphic Display Calculators)? Is it the homework assignments? Is it the variation in my teaching? Is it me, as the teacher, with my engagement, my enthusiasm, my personality or my demands? Are the students just more mature? Is it the new approaches in their thinking and learning that they developed during the academic year? Is it learning maths through English? Are there other reasons?

2.1 Purpose

When I began the project my main interest was to explore the students’ mathematical thinking and reasoning through a selection of mathematical Activities we would do over the course of three years.

The study would focus on these questions:

*Which of the Activities trigger the students reasoning and thinking?*

*How do the students see their own learning in mathematics developing from the Activities?*

*Is there anything else we are doing that affects their thinking and reasoning to a large extent?*
3 LITERATURE RELEVANT TO THIS STUDY

To me, problem solving in mathematics means that the students are active and working with a given task that is neither of standard nature nor one that should be solved by routine. The task should be possible to expand to include many qualitative levels so that all students, regardless of skill level, will be challenged to develop their math skills. Through striving to understand a concept, a procedure or a strategy, the students discuss and reflect on their thinking, thereby building the mathematical knowledge that is necessary to understand more advanced mathematics.

Through discussion, students develop a more reflective approach as well as a vocabulary suited to a discourse on mathematics. They learn more than they realize during these sessions. Researchers in Mathematics education who view problem solving in this perspective are my key sources for this project.

Professor Edward A. Silver from the University of Michigan has done extensive research for many years on the use of mathematical problem solving and problem posing. He shows that these elements are central to the discipline of mathematics and the nature of mathematical thinking. Silver (1997) wrote that through the use of inquiry-oriented mathematics instruction that includes opportunities for problem posing and problem solving, teachers can assist students to develop greater representational and strategic fluency and flexibility and more creative approaches to their mathematical activity.

Studies that have considered students’ attitudes and beliefs in different curriculum programs have also produced a fairly consistent finding: students taught using problem solving approaches are more likely to have positive and broad attitudes about mathematics (Silver 2003). Many studies show that students who have had the opportunity to work on problem-solving approaches tended to perform as well – or better – than those who worked traditionally. These large-scale curricular studies tend to mask differences in the manner in which individual teachers implement problem-solving approaches. Therefore, Silver asserts that research on problem solving should focus on the details of classroom implementation in addition to reporting outcomes associated with the adoption of particular curricula.

Professor Ole Skovsmose from the Aalborg University in Denmark has a special interest in critical mathematics education. Skovsmose (2001) thinks that traditional mathematics education falls within an exercise paradigm. This paradigm is contrasted with landscapes of investigation that
serve as invitations to students to become involved in processes of exploration and explanation. To help abandon the authorities of the traditional mathematics classroom and allow students to be the actors in their learning processes, teaching needs to move away from the exercise paradigm and in the direction of landscapes of investigation. Moving away from reference to pure mathematics toward real life references may help to provide resources for reflection on mathematics and its applications. Moulding the students into acting and reflecting learners provides mathematics education with a critical dimension.

Dr Eva Taflin from Högskolan Dalarna wrote her PhD dissertation on how to define and explore what mathematical problem solving entails. One of the most important aspects of problem solving, according to Taflin (2007), is ability to reason logically. As she saw it, this can occur through creating a discourse of mathematical didactics. Such a discourse is essential in order to develop our thinking with regard to the teaching and learning of mathematics.

3.1 Definitions

A mathematical Activity is a task given to the students that requires one entire lesson in mathematics. It can be an open problem, a rich problem or a simple problem-solving task, which may focus on understanding a mathematical concept or a procedure. It should be phrased simply so that everyone can understand the problem and, ideally, it should also be possible to extend the problem further. Some activities require a graphing calculator or similar technology.

All the students in the class should work on and discuss the same problems. The result of the Activity should always be summarized at the end of the lesson.

Discourse usually means a serious speech or piece of writing about a particular subject (Longman’s Dictionary of Contemporary English), but Sfard (2001) means that discourse is “any specific instance of communicating, whether with others or with oneself, whether predominantly verbal or with the help of any other symbolic system. It implies inclusion of instances that would probably be excluded from the category of discourse by everyday users of the term”.

An investigative approach does not yet have any established definition. Skovsmose (2001) wrote about landscapes of investigation serving as invitations to students to become involved in processes of exploration and explanation.
The term open problem has several different meanings, according to Silver (1995). There are different meanings of the term in the context of mathematical pedagogy:

- the problem itself is susceptible to different interpretations or has different acceptable answers
- the problem invites different ways to find a solution
- the problem naturally suggests other problems or generalizations
- the problem invites exploration and communication about mathematical ideas

By using these problems in mathematics education we assist the students to learn the ways of thinking and reasoning employed by those who apply mathematics and quantitative reasoning effectively to solve real-world problems, according to Silver (1995).

The concept of rich problem is not clear-cut but has different contents in different contexts. These are the seven criteria which can be used according to Taflin (2007):

- the problem shall introduce important mathematical ideas
- the problem shall be easy to understand and all students must be able to work with it
- the problem shall be experienced as a challenge, shall demand some extra effort and shall be permitted to take time
- the problem shall be able to be solved in several ways, with several mathematical ideas and ways of reasoning
- the problem shall be able to initiate a mathematical discussion based on students’ various solutions, a discussion which touches upon several mathematical ideas
- the problem shall be able to function as a bridge-builder
- the problem shall generate new and interesting problems on the part of both students and teachers.
4 METHOD

The main group of students studied in this project has been the pre-IB class at Åva Gymnasium during the academic year 2012/2013. In August 2012 the class consisted of thirty students. In June 2013 the class consisted of 26 students. There is also a smaller group of students who started the IB programme and are enrolled in the Mathematical Studies or the Standard Level Mathematics course. All of these students completed their pre-IB year at Åva.

Out of all the Activities we did during the first year in the pre-IB class I selected what I consider to be two good Activities and one less good Activity. To find out the learning outcomes for the students I used mainly these methods;

- Interviews with students from the pre-IB class which explored the students’ understanding of an Activity completed a week earlier.
- Interviews with students from the IB DP second year (IB2) who were scheduled to write their final examination in May 2013. The focus here was on how the students can account for the development of their mathematical reasoning and thinking during the years in my class.

4.1 Technology

The technology we used is the TI-Nspire Graphic Display Calculator and the TI-Nspire Navigator system that connects the calculators in a wireless network. We also used the TI-Nspire Software. The students mainly use the software to work with their homework assignments. I use it to demonstrate and visualize different elements of mathematics, when suitable. The students were able to borrow a calculator from the school during the years spent in these classes. Therefore they could use the GDC in class as well as at home.

4.2 Group size

The pre-IB class started off with 30 students. During the year some students left and others joined the class. During Activities in the autumn the class was usually split into two groups of about fifteen students. During the spring the class was less often split into two groups; instead the class was split into even smaller groups with only three students. Working in groups of three was recommended by Jonas Sjunneson and Ove Johansson (2007) in their work with “Gruppdiskussioner i matematik på gymnasiet” (Group Discussions in Mathematics in Upper Secondary Schools).
4.3 Ethics applied in the classroom

In August 2012 all students/parents/guardians participating in this research were given information about the project and asked for their consent to participate in the study. Students would remain anonymous in the final project so that it would be impossible to trace any information to a particular student.

My role as a teacher and a researcher is ambiguous. The students knew me so they may have told me more than what they might tell a stranger who visited the class a few times. On the other hand, I assess the students in pre-IB. Therefore the students could hesitate to show or explain what they really think. Maybe my two roles in this project have been more of a problem for me than for the students. Anyhow, I have tried to do my very best.

4.4 Activity one - The Gradient

The class was split alphabetically into two groups, A and B. The lesson was 90 minutes and each group met in class for 45 minutes. The aim of this activity was to introduce the concept of the gradient of a straight line. The task was for each student to contribute an illustration of a straight line through two points in the Cartesian plane on their Graphic Display Calculator, TI-Nspire. They used the device to find the gradient of the line and the coordinates of the points. All the calculators were connected through a wireless system, TI-Nspire Navigator. This enabled the display of each student’s illustration on a large screen. This sharing of work on a central screen provoked intensive discussions, guided by me, regarding how to calculate the gradient of a line.

Figure 4.4.a  Figure 4.4.b

Examples of student input with the TI-Nspire
As the discussion proceeded the students were asked to move one of their points to make a line with a different gradient or the same gradient. The new sample of contributions fueled a discussion to find out how to calculate the gradient from the coordinates observed. At this point, the idea to express the gradient as \textit{rise over run} was introduced. The class ended with a five question quiz about the gradient that I sent them through the wireless system.

At the end of the class, all students were handed an instruction sheet on how to re-create the exercise in TI-Nspire. (\textit{Appendix 1}) All students kept the file with the quiz on their calculator. (\textit{Appendix 2})

4.5  Activity two - Little Boxes

The class was split in two groups, A and B, as before. Each group was split into smaller groups of three students. The classroom was reorganized with three chairs at each table. The students entered the room and were told to take a chair in one of the formed groups in a semi-randomized order. The lesson was 120 minutes and each group met in class for 60 minutes.

The aim of the activity was to introduce the concept of functions. The task was to find out how the volume of a paper box varies and what the maximum volume could be. The box was made from a $20 \text{ cm} \times 25 \text{ cm}$ paper where identical squares are cut from the corners. Then the paper was folded into an open box. Each student made such a box in their first lesson in August and these were now assembled to help out with this activity. The students made a table of relevant data, graphed a scatter plot of the data and hence, found a function for the data in their TI-Nspire. With the help of the graph they found the maximum volume of the box. All the calculators were connected through a wireless system, TI-Nspire Navigator. The display of each student’s calculator could be projected to the white screen so everyone could follow the work.

**Figure 4.5.a  Figure 4.5.b  Figure 4.5.c**

\begin{center}
\textbf{Diagrams of the Little Boxes}
\end{center}
My task was mainly to listen to their reasoning and discussions, and to participate only to support their arguments or to discuss how they would like to visualize their data. Some groups needed more support than others to plot how volume depends on the height of the box. By watching the projected screen they could also get help from each other.

When most of the groups had finished their graphs and found the maximum point, I summarized their work by doing the same construction on my own computer. Then I posed a new problem: what if we would use a 30 cm x 40 cm paper instead? What would the maximum volume of the box be if I create a box in the same manner as before? What would the height of the box be? This was their homework for next class when I would use the new problem to introduce functions, relations, domain and range. *(The answers are 3.0×10³ cm³ and 5.7 cm).*

The students were given an instruction sheet at the end of class with a guide to how to do the exercise in TI-Nspire. See Appendix 3. Three weeks later students received a homework assignment on Little Boxes. See Appendix 4.

### 4.6 Activity three - Generalizations in Geometry

The class was split into groups that separated those who planned to continue in either Mathematical Studies or Mathematics Standard Level in the IB programme the following year. Each of these groups was split into smaller groups of three students. The classroom was reorganized so that the students could work in groups of three. When the students entered the room they were asked to fill up the formed groups in a semi-randomized order. The lesson was 120 minutes and each group received 60 minutes of instruction.

The aim of the activity was to develop a general reasoning when comparing the area between parts of different shapes. The shapes - circle, sphere and cylinder - were familiar to the students. The task was to be able to find out what part of a circle is or is not shaded when there are one or more circles inside the larger circle. We proceeded from knowing the radius of the circle to not
knowing the radius, which required the introduction of variables and abstract, general reasoning. To highlight the transition, the shape was grey when the radius was known and coloured when the radius was unknown (see figures below). What if we move from two dimensions to three dimensions? How can we reason to find out what part of a cylinder several spheres occupy?

Figure 4.6.a  Figure 4.6.b

Figure 4.6.c  Figure 4.6.d  Figure 4.6.e

Figure 4.6.f

All groups were given colour handouts of the problems. The students discussed the first problem for approximately five minutes, or until I could see that the majority of the groups were finished. Then I solved the problem with their help on the white board and we discussed it thoroughly. Then I gave them the next problem. There were five problems in all. I circulated among the groups to listen to the students’ reasoning, and interfered when I thought they needed support or further discussion.
The first problem asks what part of the big circle is shaded if the radius of the smaller circle is 3.0 cm. What happens if the radius is not known? The following problem was to find what part of the big circle is blue when you don’t know the radius of the small circle. The third problem contains more circles. The radius is 2.7 cm for the circle in the middle, and the task is to find what part of the big circle is shaded. The fourth problem is different for the two groups: either to find what part of the big circle filled with smaller circles is not orange, or to find what part of the big circle filled with smaller circles is orange. What happens if you move into three dimensions? The final problem was to find what part of the cylindrical container four tennis balls occupy.

The students received a handout at the end of the class with all the problems in black and white. (See Appendix 4) Three weeks later the students received a Quiz on Generalizations. (See Appendix 5)

**4.7 The Interviews**

There were a total of nine interviews with the students in pre-IB. The students were interviewed in pairs for 15 – 25 minutes on Monday afternoons the week after they had the lesson with the Activity. The selection of students for the interview was based on what happened in class during the Activity. I tried to get three groups with different learning outcomes to constitute the sample for each Activity.

Interview 1 concerned the lesson with the Gradient.

Interview 2 concerned the lesson with the Little Boxes.

Interview 3 concerned the lesson with the generalizations in Geometry.

The questions for the students;

1. Did you know about the concepts of “the gradient of a line”, “a function” or “generalizations, or a general reasoning” before our lesson last week?
2. How did you experience the lesson last week?
3. What do you now know about the gradient of a line, a function or how to find what part of a geometrical shape is shaded if you don’t know any measurements?
4. How do you know?
5. How would you explain the concept of “the gradient of a line”, “a function”, or “how to find out what part of a geometrical shape is shaded” to a new or an absent student who never heard about it before?

6. a) If you would choose two points, i.e. (-1, 5) and (2, -4) or (-4, 2) and (4, 6), in the Cartesian plane, then what is the gradient for a line between these two points?

b) Can you find a function for the volume of a box made from a paper (21.0 cm x 29.7 cm) if you remove four identical squares in the corner?

c) Can you find what part of the figure is shaded in this figure (a circle inscribed in a square)? Can you find what part of the cube the sphere occupies (a sphere inscribed in a cube)?

There were also interviews with three students in their second IB Diploma Programme year. They were interviewed for 20 – 25 minutes. The students were informed of the question several days ahead. They were asked if they could give an account for how their mathematical learning developed during the past three years and especially during the pre-IB year.

Interview 4 with a male student in his second year in IB DP.

Interview 5 with a female student in her second year in IB DP.

Interview 6 with a male student in his second year in IB DP.

The more specific questions were;

1. What was your attitude to Mathematics before you started the pre-IB year?

2. How did your learning in Mathematics develop during the years?

3. Which one of the Activities triggered your reasoning and thinking during the first year? How?

4. Did we do anything else during the first year which helped you to understand, think and reason better in mathematics?
5 RESULTS AND ANALYSIS OF THE INTERVIEWS

5.1 Interviews with the pre-IB students

The students were interviewed in pairs for 15 – 25 minutes on Monday afternoons the week after they had the lesson with the activity. The interviews took place in October and November 2012 and in February 2013.

5.2 Activity – The Gradient

Three groups of students with different learning outcomes constitute the sample for this activity. A total of four girls and two boys. It was the students’ results on the Gradient Quiz (see appendix 2) which formed the three different groups; not understood, partly understood and understood.

The concept of the gradient

Four students had never heard about the Gradient before this lesson.

Two students thought they had heard of and worked with simple questions concerning the gradient before this lesson, but they said it was not as advanced as in our lesson.

The lesson

All students were very positive. When I asked why, they said:

- I think it was a good lesson. I like doing things with the calculator. I think its easier learning that way, since it’s more visualized and I learn how to use the gradient of the line and how to calculate it, with algebra, just rise over run.

- I think it was really good to start out with seeing everything on the big screen and connecting all the calculators, cause then you could, first look at it, how the calculator
counted and then you could find out the formula yourself and how to do it. First you can see it on the big screen, and then you can start thinking for yourself.

*The learning outcome from the lesson*

To use a visual approach with a line to explain what the gradient of a line is was done by the four students who partly understood. They wanted to use squared paper or to have the Cartesian plane on the white board to help them find out how many steps you would need to show the *rise over run*.

To use the formula of \( m = \frac{y_2-y_1}{x_2-x_1} \) to explain what the gradient of a line is was the preferred method by the two students who understood (based on Quiz results).

*The knowledge achieved from the lesson*

All three groups needed guidance to find the correct value of the gradient. The group who understood very little (on the quiz) worked with a line through the points (-4, 2) and (4, 6). One student believed the gradient was negative and the other one said that it was positive. After some discussion they both agreed on this reasoning:

- The difference between the x-coordinate is 8, because negative four up to zero is 4 and then from zero to four is 4, so 4 plus 4 is 8, says the student. From two to six it’s 4, so the gradient is 4 over 8, so it’s 0.5.

The group who understood more (according to the quiz) used a line through the points (-1, 5) and (2, -4). Both students immediately said that the gradient is negative. When I ask why, both of them changed their mind and said that the gradient is positive. When I asked why again, they said:

- When I get an outside opinion on my answer, I start doubting my answer and start finding other answers, said the male student.

- I’m pretty sure at this but I’m not super confident. So when I hear something else, I start to doubt certain - everything, said the female student.
After some discussion they both agreed that the gradient is – 3. One of the students also emphasized that it would be easier if he would have had pen and paper instead of having to do all the calculations in his head.

The group who understood (100% on the quiz) worked with the line through the points (-1, 5) and (2, -4). One of the students thought the gradient was positive and the other student said that the gradient was negative. After some discussion they both agreed on this reasoning:

- The difference between the y-coordinates is nine, said one student.
- The difference between the x-coordinates is three, said the other student.
- So the gradient is -3, said the first student
- No, it is three, said the other student.

After some discussion she finally agreed that the gradient is -3.

5.2.1 Analysis of the interviews about the Gradient

The groups were selected based on their performance on the Gradient Quiz which was done at the end of the lesson. The results on this Quiz do not correspond to the knowledge the students showed during the interviews.

The students who wanted to use a recipe, such as the formula for the gradient, did not seem to understand how to use the numbers correctly. These students had understood completely according to the quiz.

The students who used a visual approach understood the concept of the gradient, but they lack experience working with such of problems. They need to practice more.

The students lacking confidence changed their answers and reasoning all the time, even though they understood the concept as well as the procedure.

This was the first time the majority of the students confronted the mathematical idea of the gradient of a line. Taflin (2007) says that one of the most important aspects of problem solving is the opportunity to develop the ability to reason logically. This can occur through the creation of a discourse about mathematical didactics. Such a discourse is essential in order to develop our thinking regarding the teaching and learning of mathematics.
The students who said that the gradient was a new concept for them met an idea for how to find the gradient using the method of *rise over run*. This method can be generalized to fit all real numbers so that through this reasoning they will be able to make connections and develop ideas that will be of great use to them later. Wells (1987) suggests that students, unknown to themselves, will also develop more subtle understandings that are more difficult to put into words. The interviews were made in October, when the student’s vocabulary was not yet developed enough to express their understanding properly, I think.

My reflections after this interview led to further questions: What does the student mean by knowing? How can I trust my students’ answers?

### 5.3 Activity – Little Boxes

Three groups of students with different learning outcomes constituted the sample for this interview, four girls and two boys. My observations from the lesson formed the three groups; not understood, partly understood and understood.

*The concept of a function*

All the students said they had heard about functions but were not really sure if they had used the function, at least not at this level. One student explained that he had an introduction to functions the previous year, but in French. Another student said she had done some functions but they were very simple functions and it was very long ago so she didn’t remember them really.

*The lesson*

All students were very positive. When I asked why, they said:

- I learned a lot, but I’m still trying to figure out some things. Cause I still find it difficult, like overall, it just difficult.

- I thought it was funny; it was quite easy to understand the concept once I knew what a function was. It was just interesting. I liked it because I could understand it and grasp it, without any problems.
Kind of fun, I guess. It’s not as boring as doing lots of things in the book. It’s more fun to do it in like a graphical way. It’s really fun to use the calculator and I feel that at this time I actually understood how to use it and I could really connect with the boxes and the functions and the calculator.

The learning outcome from the lesson

The students who didn’t understand can’t explain anything about what a function means to them.

The students who understood partly said:

- A function is a way of calculating the volume. It just shows the pattern which the volume increases and decreases in, on a graph. So it would be almost like an equation but it’s not. It’s not a real equation but it just shows you the pattern which or how the volume would look like and the function shows all the points.

The students who understood said that

- A function can describe the relation between x and y, and when you graph it, it does not have to be straight, but I think it can be straight. It’s just a relation between x and y, and it doesn’t have to be in a specific way to be a function. And it’s only one x for every y, or is it the other way around? Hmm, it was something like that when we worked in the book. I can’t remember exactly.

The knowledge achieved from the lesson

The function that describes how the volume varies with the height of a box made from a paper measuring 21.0 cm times 29.7 cm was answered correctly by all students, who said that the function is: x times (21 – 2x) times (29.7 – 2x) since it is (height x width x length).

The students also said that if they had the calculator they would also be able to graph the function and then find the maximum volume of the box you can create from this kind of paper.

5.3.1 Analysis of the interviews about the Little Boxes

This Activity was an introduction to the topic of functions. There were some lessons after we had done the Activity and before this interview that contained more work with functions.
Only one student could give a reasonable explanation for what a function is. The other students lacked the vocabulary to express their thinking and reasoning. The importance of this was expressed well by one student during the interview:

- I think if we did something like this more often it would be easier in the test, to explain our, well, equations and all that.
- What do you mean? Do more interviews? Should we do interviews all the time?
- To speak, to explain, I think it would improve our mathematical vocabulary.

Hodgen and Wiliam (2006) express in their principles of learning that the students must be active in the process – learning has to be done by them, it cannot be done for them – and that the students need to talk about their ideas. When students are talking about mathematical ideas they are using and constructing the language of mathematics. “Talking the talk” is an important part of learning.

It is easier to learn a procedure than to grasp a concept, I think. All students learned how to figure out a function to express how the volume varies with the height of the box. This example illustrates a process that can be applied quite generally to many mathematics problems. This is an opportunity for students to construct mathematical ideas and to refine their thinking.

5.4 Activity – The generalizations in Geometry

Three new groups of students with different learning outcomes constituted the sample for this activity, two boys and four girls. My observations from the lesson formed the three groups: not understood, partly understood and understood.

The concept of generalizations

Two students had never heard about generalizations before. Four of the students heard about them before and three of them could tell that generalizations were in topics of geometry and algebra.

The lesson

The students were positive to the lesson, especially that the class was split in two groups and then into smaller groups of three students. When I asked why, they said:
- I thought it was good and especially working in groups, because then people pointed out things that you haven’t thought of yourself, it’s easier to solve problems when we work together. And then I thought it was a good idea to go through it on the board after we tried ourselves, said one student who partly understood.

- In split class it’s like more quiet, so it’s easier to focus, said another student who understood.

One student offered another opinion about the small groups:

- In my group it was kind of, well I think everybody got it, but it was mostly, like me doing and then explaining to the others and just moving on. It depends on what group you have. This last Thursday I was in a group where everybody was in the same level that was better than this last Wednesday because then everybody wasn’t in the same level. So on the Thursday lesson, we did everything much quicker and faster.

The learning outcome from the lesson

Four of the students had never done this before. They understood the idea of having a smaller circle in a bigger circle, but to find out what part was shaded was very difficult. The explanation from the students who partly understood contains a stepwise reasoning, more like a recipe. There was no real understanding of the difference if they use numbers or letters.

To explain by showing a lot of examples is what the students who understood would like to do:

- I would explain like whatever number we put as the radius for the circles we get the same answer. So if we put like a or x we also get the same answer, one boy said.

The knowledge achieved from the lesson

To find out what part the circle occupies of the square was a task done correctly by only two of the students. The other students found the answer of $\pi/4$ or $79\%$ through some (or a lot of) guidance from me. Two of the students could find out how much the sphere occupied of the cube. The other students needed a lot of guidance to find the answer of $\pi/6$ or $52\%$.

Students reasoning stops at different levels depending on the student’s preknowledge. They needed guidance concerning the formula for the area of a circle and a square, in order to understand the problem correctly and to work with algebraic expressions.
What is the difficulty in this problem? The students explained what they struggled with like this:

- The difficult part is when I have it, like measurement, what to do with them, how to divide them, if to divide then what to do with them, says one student who partly understood.

One student said frankly after the interview:

- I didn’t like it! Because it was so, I don’t know, it was weird. Because I didn’t get it.

How can I trust my student’s answers? They were so positive and said that they learned a lot but what did they actually learn?

### 5.4.1 Analysis of the interviews about the Generalizations in Geometry

This Activity as well as the interview was a challenge for many of the students. The Activity itself is challenging and promotes thinking and discussion. Working in small groups that generate peer discussions in which the students use and construct a language of mathematics is an important part of learning, according to Hodgen and Wiliam (2006). With my guidance the students could solve the problems. Wells (1987) asserts that students want to be challenged. Then they are able to make connections and develop ideas that will be of great use to them later.

Working with algebraic expressions that contain $\pi$ is more demanding and requires a good knowledge of algebra. The students did the topics in Algebra long before they did this Activity, which is done at the end of the topic on Geometry. Perhaps an insufficient possession of knowledge of algebra is the difficulty in this problem. Learning how you can vary the conditions of the original problem is invaluable and I don’t think the students realize this yet, if they ever do. The instructional approach “What-if” fosters a creative disposition toward mathematical activity, according to Silver (1997).

Some students lacked confidence in mathematics and therefore the occasions where students confront mathematical ideas are important in building their understanding of more meaningful tasks. One student said;

- It goes well in class when I focus by myself and with the book, but then, when I leave the classroom all the confidence disappears again. I don’t know really why.
5.5 Interviews with the IB2 students

The students were interviewed for 20 – 25 minutes. The interviews took place in September 2012 and in January and March 2013.

5.6 Student One (IB2)

His pre-knowledge of mathematics was scattered and poor before he started at Åva. He did not enjoy Maths. He failed the first two tests in pre-IB Maths but finally ended the year by achieving VG in MaA and G in MaB (Swedish system). He graduated with Grade 5 in IB Mathematics Standard Level. How did he build a new relation to maths - a new foundation and understanding during his first year in pre-IB?

- My first year in pre-IB with maths was different than with any of the other years with maths I have had. It introduced me to some new things about maths that I never thought about before. I remember even the first homework assignment we had, even though I didn’t do well on it. Along with that assignment the idea of a general solution to a problem was introduced to me. And that I had never seen before. The use of letters to represent numbers to give a general solution, that was something new I had never seen before. That was the first one that really, that made me look at math differently; I guess that’s the best way to describe it.

- Then of course we had these discussions and the investigations with TI-Nspire. Some of them where - I remember, some of them distinctly - for example the one about the parabolas. I even brought the paper with me; he says and shows it to me. We were supposed to try to duplicate the patterns with parabolas. To put in the right number to make them to move around. I thought that one was really good because that one, I remember that class really well, because that class was really the first one I felt that I really understood this. I knew what I was doing. That was the first math class that I’ve had in a long time where I really understood! So I went home and I managed to duplicate all these patterns and I said to myself – I understood this! And it was that investigation that really, really, sort of, I don’t know, stimulated my brain, or I got it working.

It took him more than half a year before he felt that he had a lesson in maths where he felt so confident again. He could also tell me about other activities we did during the first year and the
common denominator for the ones he mentions is that they either contain an investigative ap-
proach or that they included discussions with generalizations.

5.6.1 Analysis of Student One (IB2)

To hear this student talk about what activities triggered his mathematical thinking and reasoning
two years ago is like listening to Skovsmose (2001) and his landscapes of investigation that serve
as invitations for students to become involved in processes of exploration and explanation. The
students are in charge in a landscape of investigation. The students are invited to formulate ques-
tions and to look for explanations. When the students take over the process of exploration and
explanation in this way, the landscape of investigation comes to constitute a new learning milieu.

Over time it was also clear that this student achieved a much more firmly based understanding as
well as a greater enjoyment and motivation. The new relationship with mathematics this student
developed during the first year can be part of a more creative approach the student develops over
time. Edward A. Silver (1997) asserts that you need to include inquiry-oriented mathematics in-
structions to foster creativity.

5.7 Student Two (IB2)

Her pre-knowledge of mathematics was poor before she came to Áva. She hated maths because
she couldn’t see any use for the subject and she didn’t get the grades she wanted. She barely
passed the three first tests in pre-IB maths, but finally achieved VG in MaA and G in MaB
(Swedish system) She graduated with Grade 6 in IB Mathematical Studies and produced an ex-
cellent maths project. How did she build a new foundation, a better understanding and begin to
enjoy maths during that first year?

She explained that everything happened gradually but it was mainly about motivation. She is
ambitious, but at the beginning of pre-IB she had a negative attitude to maths and lacked motiva-
tion. More than two years later she still remembered telling me that she hated maths.

- What’s the point of doing something that you can’t really do? But I think a turning point
  was when we did the first assignment. I found a pattern. Maybe there is something fun
  with maths? And then came the TI-Nspire, where we could do a lot of fun stuff.

She explained that the Activities themselves are motivating since they let her think that she has
abilities, that she can do more than she realized before. It was the first time math could be fun.
When I asked for an explanation of what she meant by it being fun she said:
It was partly that I understood the problem and I was able to do it. Then almost all the Activities were really interesting, but the Boxes were horrible in the beginning!

We did them during the very first lesson in pre-IB, I say.

I didn’t get it at all.

Then we did the boxes again when we introduced functions as an Activity. Then I gave you a homework assignment with the boxes. Then they returned in IB1 with the cubic functions. The last time they returned were when we did differential calculus.

Yes, I finally got it when we did differential calculus. Then I also realized that I could also use them in TOK (Theory of Knowledge)! But I’ll have a TOK presentation where I’m going to talk about how culture influences the way we think and two of the aspects I want to talk about other than culture is curiosity and motivation. So I mean this is also a good example.

When the interview was almost done, she said:

Maria, I want to say this, even though if I don’t do well in the final exam which I’m not hoping, I have learned so much, not only within maths, but so much I have discovered a lot about myself, partly because of the maths.

In what way? What do you mean?

That maybe certain things you, how do I say this? You may hate some things and it could be that why you hate it lies on that thing, it’s in you, and it’s in your attitude. You just need to change you attitude and then you will see another side of it. And it could be that you have certain abilities and it just takes one to take that step forward in order to discover it.

5.7.1 Analysis of Student Two (IB2)

This student’s motivation increased over time through the use of the Activities during pre-IB and the following years. She also said that the tests were important for her to be able to follow her progress through the test results. Studies that have considered student’s attitudes and beliefs have produced consistent findings that students taught using problem-solving approaches are more likely to have positive and broad attitudes about mathematics. (Silver, 2003)

Daring to take on a challenge is what I think motivated this student most of all. When she could understand the Activities, explore them and stretch her mind, then she started to enjoy maths. To provide occasions to confront mathematical ideas and to deal with them in a variety of ways is very important according to Taflin’s (2007) findings. By breaking down suitable problems to
work with at suitable times during the course is very helpful for motivation. Problems have to be easy and understandable but also challenge to stretch the mind. Therefore it is very important that teachers listens to students to make sure if and how the students have understood the problem they will work with. Wells (1987) explains it clearly: “By listening more to students, teachers learn more about what students know and how well they know it. By being listened to, students realise the teacher is actually interested in what they say and are thus encouraged to say more.”

Hodgen and Wiliam (2006) also emphasize that the students must be active in the process – learning has to be done by them, it cannot be done for them. This also requires that the students need to take responsibility for their studies and it does help to have the ambition of wanting something, as this student has. If she learned so much about herself through changing her attitude to maths, then what other talents might she possess?

5.8 Student Three (IB2)

He had a very good pre-knowledge of mathematics from the beginning of pre-IB, but it was not his favourite subject. He passed all tests with excellent results and achieved MVG in Ma A and Ma B. He graduated with grade 7 in Mathematical Studies and did a brilliant maths project. How did he perceive learning of mathematics through Activities during the pre-IB year?

- It was certainly very new, because to me, prior to this, mathematics had always been looking at the whiteboard while the teacher was talking and writing, and then trying to understand it. While the teachers tended to do the best they could with the white board, which isn’t a very easy thing to do, the subject often came across as bland efforts - just staring at the white board, occasionally raising your arm to ask a question, and so on. So the first time we had an Activity, I began feeling incredulous because I had never seen something like this during a math lesson before.

- Especially with the box problem. With the boxes made from the same sized sheet of paper but with varied lengths, widths and heights. All the colourful boxes, they looked more like something I would have in sewing, syslöjd, so it felt a bit peculiar at first. But then, when I made the connections, I remember this as very, very nice and captivating, to have something else to look at instead of the white board for a change. What are all these colourful boxes doing on the teacher’s desk? You know, it captivates the interest and it holds it there!
You mean just because it is different? Just because we’re not on the whiteboard, or we’re not in the book?

Yes, and it is diversity in the pursuance so I mean, the boxes are not only there to be pretty to look at, it’s an essential component, it illustrates the maximum volume of a body and how it can be obtained with a, say a sheet of paper of constant parameters, so, but at the same time it is not simply a black and white on the white board. It is more, it’s refreshing.

He summarized that he enjoyed all of the activities because they presented an opportunity to have a more relaxed maths lesson and it was diverse and it helped him to visualize certain mathematical concepts. He continued:

As much as I like the image of an eroded scholar with his nose poked in a big book, but, I mean, you can only do so much by just following conventional methods, eventually you need to depart convention and chart your own course and that’s what we have been doing here and I think it is great, because, I mean, if you don’t have diversity, you’ll end up with stagnation. So the entire class becomes customized, conditioned to monotone routine.

He noticed a change in the atmosphere in the classroom when we did the Activities which made him absorb things in a completely different manner:

I felt that it was more relaxed atmosphere overall because, normally, I mean in a rather rigid capacity of students and teachers, the atmosphere tends to be pretty subdued, everyone is working hard except for the one or two at the back that are playing with their IPhones. So, I mean, this was very fascinating and interesting, and it created a thaw, so to speak, so I mean that even the people that are normally not very keen on participating and perhaps work on their own, or just sit secluded in their own sphere of space time, kind of thawed up to this, in my opinion.

Can you explain more about the thaw during these activities?

Well, when we do these activities together it is almost as if we are not there in the capacity of student and professor, we are there in the capacity of making a new discovery together, more or less.

There is an interesting progression of his relation to mathematics, from fourth grade in primary school up to secondary school:

It’s been a paradigm shift in my case, I guess because, back then I didn’t like mathematics at all and Åke comes along and is very charismatic and very skilful in motivating students and then you come along with the same characteristics and with TI-Nspire and with the,
shall we say the box problems and all of those and the common discussions. My attitude to mathematics has changed quite a great deal and I think it is now among my favourite subjects, actually.

5.8.1 Analysis of Student Three (IB2)

The student’s complete change of attitude to mathematics is probably because of the Activities we did with problem solving. Silver (2003) reported that research suggests that the extra time, effort, and resources required to teach mathematics through problem solving effectively is well worth the effort if one's goals for mathematics education include producing students who understand mathematical concepts, are willing to tackle challenging problems, and see themselves as capable of learning mathematics.

Denying students any personal challenge beyond copying the teacher’s explanation effectively strips the work of most of its emotional meaning. It totally fails to exploit the students’ desire to be challenged, to think for themselves, to explore their world. (Wells, 1987) This student truly enjoys discussing. He has an understanding of the meaning, use and justifications of mathematical ideas required for Mathematical literacy according to Hodgen and Wiliam (2006). Could this be part of the reason for his change of attitude to mathematics? He has, partly unknown to him, developed subtle understandings that are more difficult to put into words, through working with the Activities and the discussions they lead to.

The thaw created in class might be what Hodgen and Wiliam describe as talking the talk: “Discussion in small groups enables all pupils to engage directly in discussions about the mathematical problem. By doing so, they are better able to understand the problem and they can clarify their own idea. As a result, a greater number of pupils contribute to whole-class discussions and their contributions are better articulated. “

Taflin (2007) wrote that the variation created by the teacher’s planning, and that the students had time to think for themselves, to converse with teacher and classmates, are all very important for learning. She also highlighted the importance of summarizing the work and the mathematical ideas at the end of the lesson.
6 SUMMARY/DISCUSSION

Which Activities triggered the students’ reasoning and thinking?

One student in IB2 stated that Activities with an investigative approach or those that lead to a general reasoning are better than others. Another student says that the Little Boxes activity is very good but it requires a long time to grasp it. In general the students think that all of the Activities are good and focus on different ideas, but there are no real super outstanding Activities.

The students in their third year also explain that in the classroom the Activities also help to create diversity and variation instead of monotonous routine work, and the tasks help to create a more relaxed atmosphere in the classroom.

How do the students see their own learning in mathematics developing from the Activities?

The students need time to see what impact these Activities have on their learning. Since it is a new approach to learning mathematics for the majority of the students it is not until their third year that they can reflect on learning mathematics in this manner. Students in IB2 explain that the Activities helped them to get more motivation, to let them succeed and feel satisfied with themselves, to recognize their own mathematical capabilities and to help them think, reflect and gain more mathematical knowledge.

The students in the pre-IB year could not express their reflections on their learning yet. The method of using Activities was still so new to them and they hadn’t fully grasped all the new mathematical ideas they confronted during this first year at Åva.

Is there anything else we are doing that affects their thinking and reasoning to a large extent?

The students claim that the homework assignments are important for their learning since they provide a challenge and often a new mathematical idea. These assignments consist of an investigative problem and require that each student should hand in a word-processed write up of their solution and reasoning. See appendix four.

Some students say that tests are very important for them to get feedback on their mathematical knowledge and how to improve. But Hodgen and Wiliam (2006) assert that formative assessment is much more important than the test results. Students who emphasized the importance of the
tests have been low-performing from the start, but their results improve drastically after they encountered Activities.

Many students explain that they learn so much from the discussions we have during the Activities. By listening to students, I learn more about what students know and how well they know it. By being listened to, students realise the teacher is actually interested in what they say and are thus encouraged to say more. The students learn from each other and hereby build a mathematical discourse. “If students’ interactions are to enhance learning, the communicative skills of the students must be taught” according to Sfard (2001, p. 44)

**My final reflections**

*It takes time* to develop the students’ mathematical thinking and reasoning. It is a gradual process which is helped immensely by working with well-chosen mathematical Activities on a long term basis. The students must learn to dare to take on a challenge and be responsible for their learning. They must become curious, ambitious and yearn for further knowledge and understanding. As a teacher, I must dare to work with tasks where I can’t predict what will happen, and I need to listen to the students’ reasoning and thinking.

If students are in charge of their learning because they are invited to formulate questions and search for explanations students will become more active and reflective, according to Skovsmose (2001). He also says that any landscape of investigation raises challenges for a teacher since the teacher cannot predict what question comes next. The task is to make it possible for the teacher and students to operate in co-operation within a risk zone, and to make this operation a productive activity and not a threatening experience. This means, for instance, accepting that “what if” questions can lead the investigation into unknown territory. During the last six years when I worked and developed the Activities, I learned to predict how students will approach different tasks. My skills in guiding open discussions have improved immensely over the years, and there are always nice surprises along the road.

Learning mathematics is an initiation to mathematical discourse says Sfard (2001). Through the Activities the students discuss and expand their mathematical vocabulary and understanding of new concepts and procedures. Mathematical thinking develops through correct questions, according to Taflin (2007), but I think it is essential that there be a long term plan for how and when to use the Activities. It is important to stop at times, to take the time to explore and discuss as a group and to provide an occasion to introduce a new mathematical idea. The tasks need to be meaningful and understandable. Therefore, a long term plan is needed to show how activities can be linked to all the topics in the syllabus in a logical manner.

In general all these tasks assist the students to learn new ways of thinking and reasoning in Mathematics. The students in the pre-IB class hadn’t grasped some of the activities yet but they still
provide occasions for students to be confronted with new mathematical ideas so they start to build a mathematical discourse. Taflin (2007) states that “such a discourse is essential in order to develop our thinking with regard to the teaching and learning of mathematics”.

“Students who have had the opportunity to work on problem-solving approaches tended to perform as well – or better – as those who have worked traditionally”, according to the findings of Silver (2003). He also asserts that studies have consistently found that these students are more likely to have positive and broad attitudes about mathematics. This is also in line with my findings for the students in IB2. All three students have changed their attitude to mathematics completely and recognized their own mathematical capability.

To follow up this research it would be interesting ask the question suggested by Silver (2001): What happens inside classrooms where problem-solving approaches are used effectively?
7 REFERENCES


The International Baccalaureate Organization (2012) *Mathematical Studies SL guide*


Sfard, A. (2001) *There is more to discourse than meets the ears: Looking at thinking as communicating to learn more about mathematical learning*. Educational Studies in Mathematics, 46, 13-58.


Appendix 1

The Gradient

If you define any two points in the Cartesian plane, then you can draw a line through the two points. Measuring the gradient or the slope of the line is easily done with TI-Nspire.

How can you calculate this value if you only know the coordinates of the two points?

Instructions:

1. ☐ / New document / The Graphs application

2. ☐ / View/ Hide Entry Line

3. ☐ / View / Grid / Dot Grid

4. ☐ / Geometry / Points & Lines / Point
   Click and drop a point on a grid point. Click again and drop a point on another grid point.

5. ☐ / Actions / Coordinates
   Click on the points and drop the coordinates in a suitable place.

Press esc and move the points around by pressing △ for a long time or press ctrl and △. Click again to release the point.

To draw a line between these two points;

6. ☐ / Geometry/ Points & Lines / Line
   Click in each point and then press esc

To find the gradient or the slope of the line;

7. ☐ / Geometry / Measurements / Slope
   Click on the line, press a to drop the measurement and then press esc to be able to move the points to new locations.

Investigate how the slope/the gradient of the line varies depending on where the two points are located.

How can you calculate the slope/the gradient of a line if you only know the coordinates of two points?
Appendix 2

The Gradient – Quiz

THE GRADIENT – QUIZ

There are five questions about the gradient.

\[ m = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} \]

The gradient of the line is

\( \frac{1}{2} \)

\( \frac{1}{3} \)

\( \frac{2}{3} \)

The gradient of the line is

\( -3 \)

\( -2 \)

\( 2 \)

\( 3 \)

The gradient of the line passing through the points \((-3, 4)\) and \((5, -2)\) is

\( \frac{3}{4} \)

True

False

What is the gradient of a horizontal line?

Student: type response here.

Well done!
Please wait and I will collect your files for a final discussion.

///

Maria
Appendix 3

**Little Boxes – Investigation**

Use a paper that is 20 cm by 25 cm to make a box.
Cut squares from each corner of the rectangular sheet of paper that are \( ? \) cm by \( ? \) cm.
Then fold the sides up to form an open box.

See the figure below.

How does the volume of the box depend on the height of the box?

![Diagram of a box made from a rectangular sheet of paper with squares cut from each corner.]

**Investigation:**

1. ☁️ / New document / Add the List & Spread sheet application

2. Label the columns A to D with height, length, width and volume (press Enter after the label and move to the next column head cell)

3. Fill the cells in column A with the possible heights you have worked out (\( x \in \mathbb{Z}^+ \)). Start in cell A1.

4. Work out the corresponding widths and lengths and finally the volume

To make a scatter plot to show how the volume varies with the height of the box you need a new page with a Graph application

5. ☁️ / Graphs application
6. **Graph Entry/ Scatter Plot**

Click in the entry line and write the name of your x-variable (height) and then press right arrow or TAB to write the name of your y-variable (volume). Press enter to confirm your choices.

There is not any scatter plot visible! Your window settings are not correct. You need to get a suitable scale on your x- and y-axis!

7. **Window/Zoom / Window Settings**

Choose XMin, XMax, XScale and YMin, YMax and YScale as in this screen to be able to see both the x- and the y-axis.

Can you see a pattern? What conclusion can you make?
How does the volume of the box vary with the height of the box?

This reasoning is based on an assumption that the height only can take integer values. What if the height could take any real value, x, such that \( 0 < x < 10 \)?
What would the pattern look like then?

Define a function for how the volume of the box varies with the height.

**Graph Entry / Function / Write your function**
Appendix 4

Homework Assignment

Use an A2 paper measuring 59.4 cm by 42.0 cm to make box.

Cut squares from each corner of this rectangular sheet of paper that measure $?\text{ cm by } ?\text{ cm}$.

Then fold up the sides to form an open box.

See the figure below.

1. How does the volume of the box depend on the height of the box?

2. For which height will you have the maximum volume of the box?

3. What is the maximum volume of the box?

Justify your answers by your investigation.

Write a clear investigation in Word and use the screen captures from TI-Nspire to make appropriate graphs.

There should be a title, an explanation for what you have been doing, a table with results, a scatter plot of the data and a function which shows how the volume depend on the height.

Finally, write a conclusion justified by your investigation!

Due Tuesday 15 January 2013

Good luck!
Appendix 5

Generalizations in Geometry

A. What part of the figure is shaded?

B. What part of the figure is blue (shaded)?
C. What ratio of the figure is shaded?

D. What part of the figure is not orange (shaded)?
E. What part of the figure is not orange (shaded)?

F. There are four tennis balls in a cylindrical container.

What part of a cylinder is occupied by tennis balls?
Generalizations in Geometry – Quiz

Time: 5 minutes

What ratio of the figure is orange (shaded)?

Answer:

Working